Adaptive Hybrid ARQ System Using Turbo Product Codes with Hard/Soft Decoding

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***Abstract*—The bit error rate (BER) performance of turbo product codes (TPC) has been considered extensively in the literature. However, other performance metrics such as throughput can be more informative in particular systems. In this letter, the throughput performance of hybrid automatic repeat request (HARQ) is considered using TPC with iterative hard and soft decision decoding. Monte Carlo simulation and semi-analytical solutions are developed to evaluate the throughput of HARQ-TPC system for a wide selection of codes. The obtained results reveal that the coding gain advantage of the soft over hard decoding is reduced significantly when throughput is adopted as the performance metric, and it actually vanishes completely for some codes. When adaptive coding is used, the soft decoding advantage is limited to about 1.4 dB.**

***Index Terms*—Hybrid automatic repeat request, turbo product codes, subpacket systems, error detection, iterative decoding.**

I. Introduction

TURBO product codes (TPC), alternatively referred to as block turbo codes, are powerful forward error correction (FEC) codes that can provide high coding gain [1]. TPC are constructed by serially concatenating two linear block codes separated by an interleaver [2]. TPC support a wide range of codeword sizes and code rates, and they are included in some recent communication standards such as the IEEE-802.16 for fixed and mobile broadband wireless access systems [3] and the IEEE-1901 for broadband power line networks [4].

The ultimate coding gain of TPC is achieved by performing a number of soft-input soft-output (SISO) iterative decoding processes that are applied to each row and column in the codeword matrix, which requires considerable computational power [5]. Consequently, reducing the computational complexity of TPC has received significant attention in the literature as reported in [5], [6] and the references listed therein. The computational complexity constraint of TPC becomes even more severe for systems that employ automatic repeat request (ARQ) protocol because particular packets have to be retransmitted, and hence decoded several times. The techniques that employ both FEC and ARQ are usually referred to as hybrid ARQ (HARQ) [7].

In the literature, Al-Dweik *et al.* [5], [8], [9] proposed new techniques to reduce the SISO decoders complexity by improving the bit error rate (BER) performance of the less complex TPC decoders, namely the hard-input hard-output (HIHO) decoders. Although such techniques managed to reduce the BER gap between SISO and HIHO decoders, the SISO BER remains considerably smaller. For example, the SISO extended Bose-Chandhuri-Hocquenghen (eBCH) (32, 21, 6)^2 and (64, 51, 6)^2 have a coding gain advantage of more than 2 dB over the HIHO ones. The gap becomes larger with higher code rates as in the case of the eBCH(32, 26, 4)^2 and eBCH(128, 120, 4)^2, where the coding gain difference surges to about 4 dB [5]. Consequently, the low complexity might not be sufficient to justify adopting HIHO decoding for practical systems due to the high coding gain penalty.

In general, most of the work considered in the literature aimed at minimizing the computational complexity under fixed BER constraint [5], [10]. However, the BER is not necessarily sufficient to describe the quality of service (QoS) for systems that incorporate HARQ, where the throughput [7] or delay [11] are more desired performance metrics. Other performance metrics such as the information outage probability (IOP) are considered in the literature as well [12]. However, mapping the IOP to QoS metrics is not straightforward.

Unlike the previous work reported in the literature, this letter considers the computational complexity performance under throughput rather than BER constraints. The computational complexity and throughput are evaluated and compared for TPC-HARQ systems using SISO and HIHO decoding. Extensive Monte Carlo simulation and semi-analytical results are produced for various TPC codeword sizes and rates. The comparison is then extended to include adaptive HARQ systems with the objective of maximizing the system throughput. Surprisingly, the obtained results reveal that HIHO decoding can offer throughput that is equivalent to SISO decoding for particular codes and signal-to-noise ratios (SNRs), which allows for significant computational complexity reduction. Moreover, the staircase behavior of the TPC-HARQ implies that significant power saving can be achieved if a suitable optimization criterion is incorporated.

II. Adaptive TPC-HARQ System Model

Consider that the information bits sequence $\mathbf{d} = [d_1, d_2, \cdots, d_K]$ is to be transmitted over a TPC-HARQ system, $d_i \in \{0, 1\}$. First, the sequence $\mathbf{d}$ is divided into L equal and independent parts $\mathbf{d} = \left[\mathbf{d}^{(1)}, \mathbf{d}^{(2)}, \cdots \mathbf{d}^{(L)}\right]$, where $\mathbf{d}^{(i)} = \left[d_1^{(i)}, d_2^{(i)}, \cdots d_m^{(i)}\right]$ and $m = K/L$. Then, the ith data block $\mathbf{d}^{(i)}$ is applied to a cyclic redundancy check (CRC) encoder where l_c bits are appended at the end of the data sequence for error detection

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purposes. The CRC encoder output can be written as $\mathbf{c}^{(i)} = \left[d_1^{(i)}, d_2^{(i)}, \cdots, d_m^{(i)}, c_1^{(i)}, c_2^{(i)}, \cdots, c_{l_c}^{(i)}\right]$, which is then applied to a TPC encoder that appends l_p bits to $\mathbf{c}^{(i)}$ as described in [1]. Each of the encoder output $\mathbf{C}^{(i)}$, $i \in \{1, 2, \cdots, L\}$ is considered as a subpacket and the L codewords are considered as one packet $\boldsymbol{\mathcal{C}} = \left[\mathbf{C}^{(1)}, \mathbf{C}^{(2)}, \cdots, \mathbf{C}^{(L)}\right]$. The packet size S_p is considered to be fixed regardless of m, K or L [7]. Therefore, the packet size $S_p = L \times S_s$, where the subpacket size $S_s = n^2 = m + l_c + l_p$. Although only one single CRC operation can be used for the entire information bits block, the throughput may degrade significantly because an error in any subpacket results in the retransmission of the entire packet [13]. The packet $\boldsymbol{\mathcal{C}}$ is then modulated using binary phase shift (BPSK) modulation and transmitted through a channel that introduces additive white Gaussian noise (AWGN). It is worth noting that there are some error detection techniques that does not require CRC bits, which might improve the system throughput [14]. However, the advantage of such approach is noticeable only for very small packet sizes.

At the receiver side, the received packet is split into L subpackets that will be decoded and checked for errors independently. If all subpackets are error free, an acknowledgment (ACK) is sent to the transmitter to proceed with the transmission of the next packet. Otherwise, a negative acknowledgment (NACK) is sent to instruct the transmitter to retransmit the erroneous subpackets. The retransmitted packets are combined using maximal ratio combining (MRC) to enhance the SNR after each retransmission. It is worth noting that MRC is not optimal in HARQ systems [15]; however, the difference between the optimal and MRC is not significant. The process is repeated until all subpackets are error free, or the maximum number of transmissions M is reached. Therefore, the NACK requires L bits to represent the subpacket number. If the number of retransmissions is equal to M, the erroneous subpackets are dropped. As it can be noted, more than one subpacket can be sent in one retransmission process, which is beneficial to reduce the delay.

The decoding of each subpacket is performed using SISO or HIHO decoding [1], [8], [9]. The adaptation process is performed to maximize the system throughput by selecting a particular L and TPC based on the channel conditions, which is the SNR for the considered system.

III. Throughput Analysis

The transmission efficiency or throughput η, is the ratio of the number of information bits received successfully to the total number of transmitted bits. Given that N subpackets are transmitted, then,

$$\eta = \frac{mz_1 + mz_2 + \cdots + mz_N}{S_s\rho_1 + S_s\rho_2 + \cdots + S_s\rho_N} = \frac{m\sum_{i=1}^{N} z_i}{S_s \sum_{i=1}^{N} \rho_i} \tag{1}$$

where $1 \leq \rho_i \leq M$ is a random number that represents the total number of transmitted subpackets and z_i is a random number that indicates if a subpacket is dropped, $z_i = 0$ if the ith subpacket is dropped, and 1 otherwise. However, because $z_i \in \{0, 1\}$, then $\frac{1}{N}\sum_{i=1}^{N} z_i$ is just the ratio of the non-zero elements to the total number of transmitted packets, i.e., the complement of the subpacket drop rate. Given that $N \to \infty$, by the law of large numbers $\frac{1}{N}\sum_i \rho_i \to \mathbb{E}\{\rho\}$ and $\frac{1}{N}\sum_{i=1}^{N} z_i \to (1 - P_d)$, where $\mathbb{E}\{.\}$ denotes the expected value and P_d is the subpacket drop probability. Therefore (1) can be written as

$$\eta = \frac{1}{\mathbb{E}\{\rho\}} \frac{m}{S_s}(1 - P_d). \tag{2}$$

For regular ARQ systems, $P(\rho = \ell) = P_s^{\ell-1}(1 - P_s)$ and $\mathbb{E}\{\rho\} = \sum_{i=1}^{\infty} i \; P(\rho = i) = 1/(1 - P_s)$, where P_s is the probability of subpacket error [16, P. 1160]. However, with packet combining, the value of P_s changes as a function of the transmission round index. Hence,

$$P(\rho = \ell) = P_s^{(1)} P_s^{(2)} \cdots P_s^{(\ell-1)} \left[1 - P_s^{(\ell)}\right] = \left[1 - P_s^{(\ell)}\right] \prod_{i=1}^{\ell-1} P_s^{(i)} \tag{3}$$

and

$$\mathbb{E}\{\rho\} = \sum_{i=1}^{\infty} i \; \left[1 - P_s^{(i)}\right] \prod_{j=1}^{i-1} P_s^{(j)} \tag{4}$$

where $P_s^{(i)}$ is the subpacket error probability during the ith transmission round. However, in practical HARQ systems, the number of transmissions is limited to M, which results in a truncated HARQ. The probability mass function (pmf) of the truncated ρ (denoted as ρ_T) becomes

$$P(\rho_T = \ell) = \begin{cases} P(\rho = \ell), & \ell \in \{1, 2, \cdots, M-1\} \\ 1 - P(\rho < M), & \ell = M \\ 0, & otherwise \end{cases} \tag{5}$$

where $P(\rho < M) = \sum_{i=1}^{M-1} P(\rho = i)$. After some straightforward manipulations, $\mathbb{E}\{\rho_T\}$ can be expressed as,

$$\mathbb{E}\{\rho_T\} = M - \sum_{i=1}^{M-1} [M - i] \left[1 - P_s^{(i)}\right] \prod_{j=1}^{i-1} P_s^{(j)}. \tag{6}$$

Consequently, computing $\mathbb{E}\{\rho_T\}$ analytically requires the knowledge of $P_s^{(\ell)}$ for $\ell = 1, 2, \ldots, M$. However, since the packet combining effect is limited to the enhancement of the overall SNR, which is denoted as $SNR^{(i+1)}$ and can be computed recursively,

$$SNR^{(i+1)} = SNR^{(i)} + SNR^{(1)}, \; i \in \{1, 2, \ldots, M-1\} \tag{7}$$

where $SNR^{(1)} = SNR$. Thus, $P_s^{(i+1)}$ can be computed as

$$P_s^{(i+1)} = P_s^{(1)}|_{SNR = SNR^{(i+1)}}, \quad i \in \{1, 2, \ldots, M-1\}. \tag{8}$$

Therefore, $P_s^{(i+1)}$ is equal to $P_s^{(1)}$ except that SNR is replaced by $SNR^{(i+1)}$. Unfortunately, computing $P_s^{(1)}$ analytically is difficult because the TPC error correction capability depends on the error pattern rather than the number of errors [5]. However, the result obtained in (6) can be used to derive a semi-analytical solution (SAS) given that $P_s^{(1)}$ is obtained via simulation. The main advantage of the SAS is that it can be

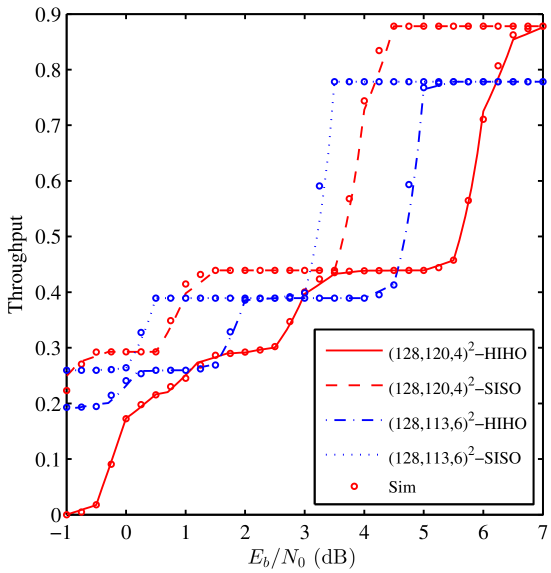

Fig. 1. Semi-analytical and simulated throughput of SISO and HIHO TPC-HARQ versus E_b/N_0.

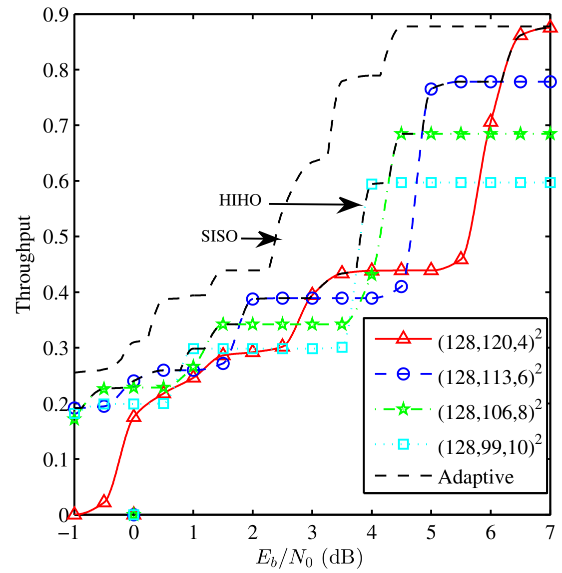

Fig. 2. Throughput of HIHO TPC-HARQ using the optimal TPC codes.

used to compute η without the need to simulate the complete TPC-HARQ system. Instead, only $P_s^{(1)}$ is needed, which is just the packet error probability of the regular TPC system. Consequently, the simulation time can be reduced remarkably.

To complete the SAS of η, the packet drop probability P_d can be computed by noting that a packet is dropped if the first M transmissions fail. Thus,

$$P_d = \prod_{i=1}^{M} P_s^{(i)}. \tag{9}$$

IV. Numerical Results

In this section, the performance of the TPC-HARQ system is evaluated in terms of throughput and complexity. The system is simulated for a packet size $S_p = 128^2 = 16{,}384$ bits. The packet is divided into L subpackets each of which is TPC encoded with code $(n, k, d_{min})^2$, i.e. $L = S_p/n^2$. The component codes used are the eBCH with values of $n = 128$, 64, 32, 16, and 8, and using all possible values of k that gives code rates larger than 0.25. Subsequently $L = 1$, 4, 16, 64, or 256. The maximum number of transmissions allowed is $M = 4$. For each simulation run 1000 packets are transmitted. The SISO and HIHO decoders are configured to perform a maximum of four iterations. Moreover, the number of reliability bits for the Chase decoder [17] is set to 4 in the SISO decoder [1].

The throughput results for the TPC-HARQ using the eBCH $(128, 120, 4)^2$ and $(128, 113, 6)^2$ is given in Fig. 1. As it can be noted from the figure, the throughput of the SISO and HIHO decoding for the $(128, 113, 6)^2$ is approximately equal for $E_b/N_0 \gtrsim 5$ dB, and in the range from 2 to 3 dB, which is remarkably different from the BER performance for these codes. The $(128, 120, 4)^2$ exhibits a similar behavior except that it is for a smaller range of E_b/N_0.

The throughput of the HIHO TPC-HARQ is shown in Fig. 2 for TPC codes with $n = 128$ and $k = 120$, 113, 106, and 99.

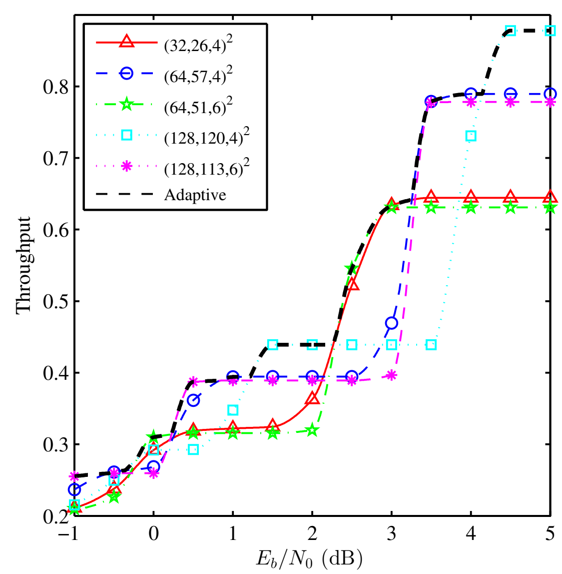

Fig. 3. Throughput of the TPC-HARQ using SISO decoding.

Although the results were obtained for various values of n and k, the listed codes are the ones that contribute to the maximum throughput in the adaptive system. Hence, the results for all other codes are omitted. As it can be noted from the figure, the code with the highest rate provides the maximum throughput at high SNR. However, TPC with smaller code rates become more efficient at lower SNRs. It is worth noting that the cross-over cycle between these codes repeats itself every 3 dB, which is due to the MRC process.

The throughput of the TPC-HARQ using SISO decoding is presented in Fig. 3. The figure is generated in a similar fashion to Fig. 2 except that SISO decoding is used. As it can be noted from the figure, the codes that contribute to the maximum throughput are not limited to the $n = 128$ family as in the HIHO case.

By comparing the throughput of the adaptive HIHO and SISO systems, it can be observed that the average E_b/N_0 shift between the two curves is about 1.4 dB.

To compare the complexity of the TPC-HARQ system with HIHO and SISO decoding, the simulation time is measured for both systems under identical operation conditions. The simulation time in hours is given in Fig. 4 as a function of E_b/N_0. The TPC decoder stops whenever the decoder converges to the correct codeword as described in [5], otherwise it completes 4 full iterations. As it can be noted from the figure, increasing the number of subpackets L for the SISO TPC-HARQ increases the complexity significantly, which is due to the fact that the complexity of SISO TPC decoders is dominated by the number of soft decision decoding operations performed for each row and column rather than the size of the component codeword. The figure also shows that the complexity of the HIHO-TPC is substantially smaller than the SISO for low and moderate SNRs. For high SNR the difference shrinks as the SISO decoder stops mainly after the first half iteration [5]. However, the SISO still requires much longer simulation time, which ranges from 2 to 7 times that of the HIHO based on the code used.

Therefore, the penalty of adopting HIHO TPC-HARQ is not as high as suggested by the BER results. Moreover, the processing power required by the SISO decoder is much higher than the HIHO decoder. Consequently, if the two systems are compared under tight power budget constraints, the throughput difference will even be less that 1.4 dB.

V. Conclusions

An adaptive subpacket HARQ scheme is proposed in this letter. The proposed HARQ is based on TPC with HIHO and SISO decoding. The adaptation process is performed to maximize the system throughput by changing the subpacket size and code rate based on the channel SNR. Extensive Monte Carlo simulation and semi-analytical results were used to compare the performance of HIHO and SISO systems with and without adaptation. The results obtained demonstrated that the throughput performance of HIHO and SISO decoding is drastically different as compared to BER performance where the throughput exhibited a staircase shape. The staircase throughput implies that power adaptation should be used as it can reduce the power consumption while maintaining the throughput unchanged. Alternatively, code adaptation can be used as well, which is usually used any way to maximize the throughput. The obtained results show that the HIHO is only ~ 1.4 dB less than the SISO when the throughput is used as the performance metric.

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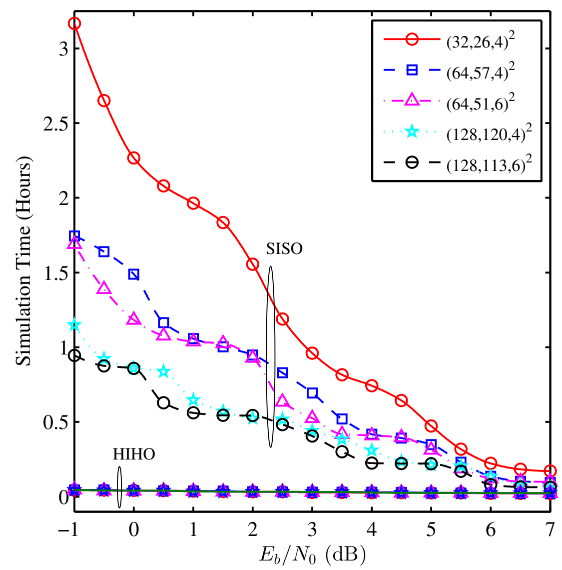

Fig. 4. The simulation time of SISO and HIHO TPC-HARQ.

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