Delay Analysis of Ethernet Passive Optical Networks with Quasi-leaved Polling and Gated Service Scheme

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***Abstract*—*Ethernet passive optical networks* (EPONs) have emerged as a promising solution for the next generation access networks. As this technology matures, intensive research work is underway to enhance its functional capability and economic viability. This work evaluates the performance of EPONs from the analytical perspective. Specifically, this study elaborates an analytical framework to explore the performance characteristic of EPONs with quasi-leaved polling operation and gated resource allocation policy. By investigating the temporal relationship of multiple events successively occurred in such a system, a graphical presentation is developed to facilitate more quantitative analysis. Using classical queuing theory, we then derive closed-form expression for the average packet queuing delay and average queue length of the researched EPON model. Simulation experiments show that the derived analytical expression can precisely evaluate the network performance for memoryless traffic inputs, as well as to closely estimate the performance of lightly loaded network for bursty input traffic profiles.**

I. INTRODUCTION

Ethernet passive optical network is viewed by many as a promising solution for the next generation access network. With optical speed Ethernet frames, maintenance-less *passive optical network* (PON) architecture, as well as newly standardized *quality-of-service* (QoS) functionalities, EPON is capable of delivering bundled voice and data services along with video broadcast over the same high-speed infrastructure in a cost-effective way.

A PON is basically an *optical line terminal* (OLT) residing in the *central office* (CO) connected to multiple *optical network units* (ONUs) near subscribers' premises. In this configuration, all ONUs share the same point-to-point uplink transmission channel based on *time division multiple access* (TDMA) technique. Due to the non-uniform behavior of traffic generated in *local area networks* (LANs), *dynamical bandwidth allocation* (DBA) is viewed as more bandwidth-efficient over static allocation schemes [1] [2]. DBA is currently a quite popular research topic of EPON technology and has been extensively studied by many researchers. However, due to the high complexity of network behavior introduced by various DBA schemes, analytical study of EPON system has not been well developed so far. As a tributary contribution to the field of EPON modeling and analysis, in this study we evaluate the statistical performance metrics of a simple EPON system that applies *quasi-leaved* polling operation and gated DBA scheme for resource allocation.

The rest of this paper is organized as follows. Section II introduces related background knowledge and reviews some notable research work. Section III exposits our proposed analytical framework based on memoryless traffic inputs. Section IV compares the mathematical analysis with simulation and visualizes the deviation of our analysis from the performance driven by bursty traffic sources. Finally, Section V presents concluding remarks on this study.

II. BACKGROUND KNOWLEDGE AND RELATED RESEARCH

A. MPCP and EPON MAC operations

In order to enable efficient statistical bandwidth multiplexing in EPON architectures, the IEEE 802.3ah has devised the multipoint control protocol (MPCP) [3]. MPCP defines a message-based control mechanism to facilitate real-time information exchange between the MAC peers at the OLT and each ONU. Two messages are involved in the regular operation mode of EPONs, i.e., *REPORT* message and *GATE* message. The *REPORT* message is sent by each active ONU and received by the OLT, to update the OLT's perception on the amount of buffered traffic at the ONU. The *GATE* message is sent by the OLT and received by the destined ONU, to inform the ONU with its granted transmission window size and upcoming transmission start time. The assignment of transmission window is based on the polling result provided by the *REPORT* message of each ONU. According to the DBA schemes studied in the literature, this transmission window can be assigned in different ways. For example, it could be assigned by:

A1 *Gated service scheme*—the granted transmission window for polling cycle n exactly matches the requested window by the ONU in polling cycle $n-1$, i.e., $G_i(n) = R_i(n-1)$, where $G_i(m)$ and $R_i(m)$ are the granted window size to ONU i and the requested window size by ONU i in cycle m, respectively.

A2 *Limited service scheme*—the granted transmission window is upper-bounded at a fixed value

W_{max} with no over-granting, i.e., $G_i(n) = \min[R_i(n-1), W_{max}]$.

$A3$ *Credit-based service schemes*—the granted transmission window is upper-bounded at W_{max} and permits certain amount of over-granting above the requested window by the ONU, i.e., $G_i(n) = \min[R_i(n-1)+\delta, W_{max}]$. The value of δ can be a constant value or be proportional to the ONU's request, i.e., $\delta = \alpha R_i(n-1)$, where α is a design parameter.

$A4$ *Non-guaranteed elastic service scheme*—the granted transmission window is not constrained by W_{max} but depends on the total amount of grants issued by the most recent $N-1$ *GATE* messages when $G_i(n)$ is computed, denoted as $\phi(i)$, i.e., $G_i(n) = \min[R_i(n-1), N \times W_{max} - \phi(i)]$, where N is the number of active ONUs in the network. This scheme does not provide any guaranteed grant to the ONUs.

$A5$ *Guaranteed elastic service scheme*—the granted transmission window is first guaranteed by W_{max}. Above this guaranteed grant, ONUs requesting larger transmission window than W_{max} can share the available resource spared by other ONUs requesting less than W_{max}, i.e.,

$$G_i(n) = \begin{cases} R_i(n-1), & R_i(n-1) \le W_{max} \\ W_{max} + \nu_i(n), & R_i(n-1) > W_{max} \end{cases} \quad (1)$$

where $\nu_i(n)$ is the extra allocation issued to heavily loaded ONU i by sharing available resource from lightly loaded ONUs.

The polling operation is also diverse in the literature. Specifically, it can be categorized into three classes as:

$P1$ *Separated polling*—the ONUs are polled and allowed to transmit one after the other, with a complete round-trip message walking overhead time required for each ONU. This scheme provides the most up-to-date bandwidth need information to the OLT before each ONU is allowed to transmit. However, as we can see, this scheme requires large signaling overhead withdrawn from the scarce network resource, especially when the OLT is serving a large number of ONUs.

$P2$ *Quasi-leaved polling*—the polling messages, i.e., *GATE* messages, for each ONU are broadcast sequentially to the downlink at the beginning of each service cycle, i.e., a service round where each ONU is serviced one time. Upon receiving and processing these polling messages, each ONU is allowed to transmit before the polling response message, i.e., *REPORT* message, of the previous ONU arrives at the OLT. This scheme permits the *GATE* messages concurrently pass through the downlink channel and uplink data burst to be concatenated with the previous *REPORT* message in the uplink channel, thereby to reduce signaling overhead and shrink the service cycle. However, the initiation of *GATE* messages in service cycle n is still contingent on the complete reception of a *REPORT* message from each ONU in service cycle $n-1$.

$P3$ *Interleaved polling*—the *GATE* message of service cycle n for each ONU is initiated by the OLT, upon receiving the *REPORT* message of service cycle $n-1$ only from this ONU. This scheme allows the downlink *GATE* messages and uplink data bursts as well as corresponding *REPORT* messages to coexist in the signal propagation "pipe". Therefore, it offers the most efficient usage of the wavelength resource among the three polling schemes, especially when the network is highly loaded.

Since distances between the OLT and ONUs may vary, the data bursts from different ONUs may collide if the transmission of each ONU is scheduled with the same time reference. To avoid this collision, the standard defines *ranging* process, where the OLT updates its perception on each ONU's round trip propagation time via every *GATE-REPORT* message exchange with the ONU [3]. Being aware of the message propagation time to each ONU, the OLT therefore can adjust the scheduling of each ONU's transmission start time accordingly, such that data bursts from unevenly distanced ONUs are ultimately lined up without collision at the receiver of the OLT.

In order to effectively delimit data bursts from neighborly scheduled transmission windows, a guard time is defined before the start of each transmission window. This guard time permits an instant break, thereby the transmitters of ONUs can be completely turned on/off, and the OLT can adjust its receiving power threshold to detect signals from unevenly distanced ONUs.

B. *Related research work*

Many of the existing research works of EPON technology focus on the DBA design and transmission scheduling algorithms. Most notably, [1] addresses many detailed issues on the design of an EPON network. In this paper the authors proposed an *interleaved polling with adaptive cycle time* (IPACT) scheme. Besides interleaved polling, i.e., $P3$ discussed above, multiple DBA schemes are proposed and quantitatively evaluated. Particularly, coupled with $P3$, DBA schemes $A2$, $A3$ and $A4$ as detailed in Sec. II-A are tested by simulations, in terms of average packet service delay. Nevertheless, IPACT is a *single service class* (SSC) based design framework, where packets are serviced on *first come first serve* (FCFS) basis through the only packet queue in each ONU's MAC buffer.

Following IPACT, other service-centric and more sophisticated DBA schemes and scheduling design are proposed in the literature. For example, in [4] the authors extended the SSC design of EPON into *multiple service class* (MSC) case, where multiple FCFS queues with different service priority feed the shared uplink channel. The authors examined the

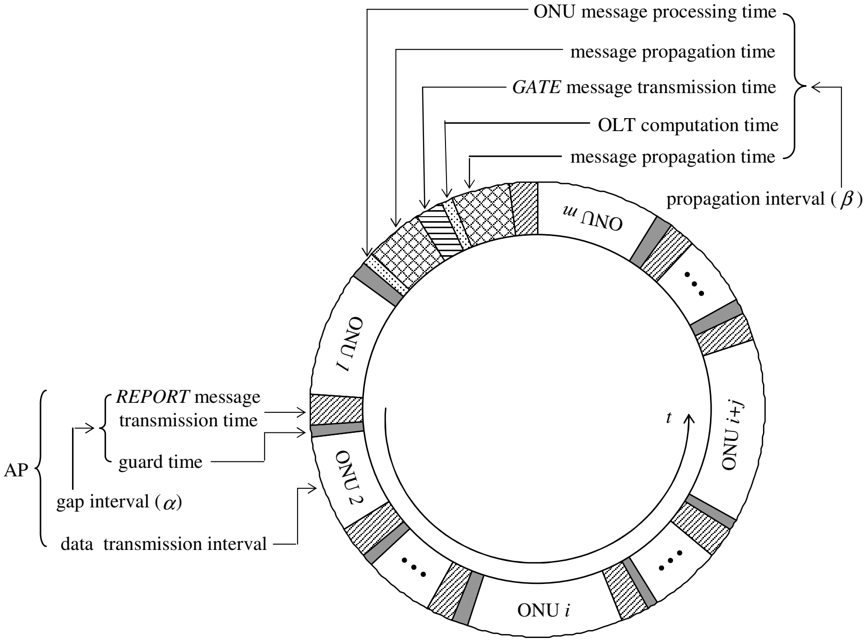

Fig. 1. Graphical presentation of sequential events in one service cycle

limited service scheme with interleaved polling for MSC, i.e., $A2 + P3 + MSC$, and noticed the *light load penalty* problem. Moreover, the authors of [2] proposed DBA1 and DBA2 schemes, where the former applies $A5 + P2 + MSC$ and the latter improves it utilizing a hybrid version of $P2$ and $P3$, according to the taxonomy discussed in Sec. II-A. In [5], the authors presented a new perspective of DBA, i.e., two-layer DBA (TLBA), where the total available bandwidth is allocated among different classes first, then among ONUs for intra-class allocation. This TLBA can be categorized as applying $A5 + P2 + MSC$.

Along with many simulation based studies, in [6], the authors developed a mathematical model to estimate the delay performance offered by the proposed *bandwidth guaranteed polling* (BGP) design, which can be categorized as $A2 + P1 + SSC$. Another good performance analysis study of EPON network is proposed in [7]. Here the authors developed a recursive model for the IPACT scheme suggested in [1]. Based on this recursive model the authors derived close-form expression of the average queue size at the ONU, for both single-ONU and multiple-ONU network setups. For the single-ONU case as well as the case of multiple-ONU with small load-distance ratio, where the latter can be understood as multiple single-ONU networks operating concurrently, the analytical model can be categorized as $A1 + P1 + SSC$. For the case of multiple-ONU with large load-distance ratio, the analysis falls into $A1 + P3 + SSC$.

In this study we consider the EPON system as a complete functional module that delays the proceeding of an arriving packet, with the intention of utilizing some well established approach for queuing analysis [8]. Specifically, we present an analytical framework on the delay performance of a SSC based EPON system applying quasi-leaved polling and gated service scheme for bandwidth allocation, i.e., $A1+P2+SSC$. Moreover, we also apply the following assumptions for the proposed analysis:

1) Packet arrivals to each ONU's MAC buffer follow Poisson distribution, with the average arrival rate of $\lambda/m\, packets/second$, where λ and m are the average arrival rate to the entire network and the number of ONUs in the network, respectively. Namely, the network load is equally distributed over each ONU.
2) Packet size is independent from the packet arrival process and is uniformly distributed between 64 and 1518 bytes, i.e., the minimum and maximum size of an Ethernet frame.
3) The network is not overloaded and therefore a steady state exists.
4) The buffer size at each ONU is large enough to contain the backlogged traffic, i.e., no packet dropping.
5) ONUs are separated from the OLT with the same distance, which can be virtually achieved through the periodic *ranging* process as discussed in Sec. II-A.
6) Message propagation time from the OLT to a certain ONU does not change with time[1].

III. PROPOSED ANALYTICAL MODEL

In order to facilitate later discussion, we first employ a graphical presentation as shown in Fig. 1 to illustrate the sequential events occurred in one service cycle by the quasi-leaved polling operation $P2$. Here, we denote the ONU scheduled for the earliest transmission within a service cycle

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[1] In practical systems, this value may vary due to small deviation of fiber refractive index resulted from temperature drift [4].

as ONU 1 and similarly, ONU scheduled for the second transmission as ONU 2, and so forth. As shown in the figure, the graph organizes the key events occurred in one service cycle into a circle that reveals the temporal relationship of these events. Specifically, at the beginning of a service cycle, the OLT sends each ONU a *GATE* message. Since we apply uniform OLT-ONU distance, i.e., equal message propagation time, the *GATE* message destined to ONU 1 is transmitted first to the downlink channel[2]. When ONU 1's *GATE* message is transmitted, after the downlink message propagation time ONU 1 receives this message and processes it with corresponding message processing time. Followed by a guard time, as shown in Fig. 1, ONU 1 starts transmitting. When the granted bytes have been transmitted, the last portion of the transmission window is used to transmit the *REPORT* message. During the transmission of ONU 1, other ONUs are continuously receiving *GATE* messages broadcast through the downlink and processing the one destined to respective ONU. Since the time required for ONU 2 to receive its *GATE* message from the downlink equals the time for ONU 1 to transmit a *REPORT* message into the uplink (*GATE* message and *REPORT* message are both 64 bytes long), even if ONU 1 only transmits a *REPORT* message with no data packet in its granted transmission window, ONU 2 will not miss its transmission start time, which is informed by the second *GATE* message and scheduled a guard time later than the termination of ONU 1's transmission. Namely, the arrival of ONU 2's *GATE* message will never be late and entail extra overhead time between consecutive transmission windows. Hence, for the same reason, we can conclude that every ONU is ready for transmitting after the guard time preceding its scheduled transmission start time. Now immediately after ONU 1's transmission, followed by a guard time, ONU 2 starts transmitting, and so forth. When the ONU that is scheduled for the latest transmission in this service cycle, i.e., ONU m, has completed transmitting its *REPORT* message, the system has to wait for this message to propagate from ONU m to the OLT through the uplink channel. Upon receiving this *REPORT* message, a computation time is required for the OLT to finish processing all of the received messages from ONUs and to perform window assignment for the next service cycle. After this computation time, the OLT then sends *GATE* messages sequentially again into the downlink channel, to initiate the next service cycle.

We define the following terms in the graphical presentation of Fig. 1 to express our analysis more concisely:

1) α–gap interval, the aggregated amount of time consumed by a guard time and its preceding *REPORT* message transmission time;
2) β–propagation interval, the aggregated amount of time consumed by the round trip propagation time, OLT computation time, ONU 1's *GATE* message transmission time and ONU message processing time (see Fig. 1);
3) φ–the total overhead time involved in one service cycle, i.e., $\varphi = m\alpha + \beta$;
4) *Arrival period* (AP)–the arrival period of ONU i is the time duration consisting of ONU i's data transmission interval and the preceding gap interval (and the propagation interval for $i = 1$);
5) *Forward neighbor* (FN)–ONU i's j^{th} FN is ONU $i + j$ modulo m ($1 \le i \le m$ and $0 \le j \le m - 1$). For example, ONU 2 is the first FN of ONU 1 and the latter is the first FN of ONU m as well as the 0^{th} FN of itself.

A. Average packet queuing delay

Consider the case where an arbitrary packet ϵ arrives during ONU i's AP at its j^{th} FN. The queuing delay of this packet includes three components:

1) The remaining transmission time of the packet being serviced (but not in the queue) at ONU i, or the remaining time of the overhead portion in ONU i's AP, when packet ϵ arrives. Namely, this is the waiting time before the service of ONU i's head-of-line (HOL) packet in the queue can be started. We denote this part as R.
2) The transmission time of all packets that will be transmitted before packet ϵ, after the service of HOL packet in ONU i's queue is started. These packets may belong to any ONU in the system. We denote this part as T.
3) The total duration of overhead time (gap intervals and propagation intervals) that will occur before packet ϵ is transmitted. We denote this part as G.

Therefore, the queuing delay of packet ϵ, denoted as D, is represented by the following equation:

$$D = R + T + G \tag{2}$$

When the system reaches steady state, the expected value of packet ϵ's queuing delay is then given as:

$$E(D) = E(R) + E(T) + E(G) \tag{3}$$

where $E(\cdot)$ denotes the expected value of the operand, i.e., the ensemble average. Since the arrival process is ergodic, time average and ensemble average in this case are interchangeable. We neglect this terminological difference in the following discussion.

In steady state, packet ϵ will see the same average number of packets queued in the system (not in service), both when it is enqueued and when it is dequeued. Since the expected number of packets arrived at the system during packet ϵ's queuing time is $\lambda E(D)$, the expected number of packets transmitted during T is also $\lambda E(D)$. If let $\overline{X}$ denote the average transmission time of a packet, noting the independency of arrived packet size from the arrival process, the expected value of T is then given as:

$$\begin{aligned} E(T) &= \lambda \times E(D) \times \overline{X} \\ &= \rho E(D) \end{aligned} \tag{4}$$

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[2] Note that if the OLT-ONU distances are various, some *GATE* messages destined to other ONUs should be transmitted ahead of the one destined to ONU 1. However, this only incurs a fixed amount of extra overhead time in Fig. 1 without more analytical complexity.

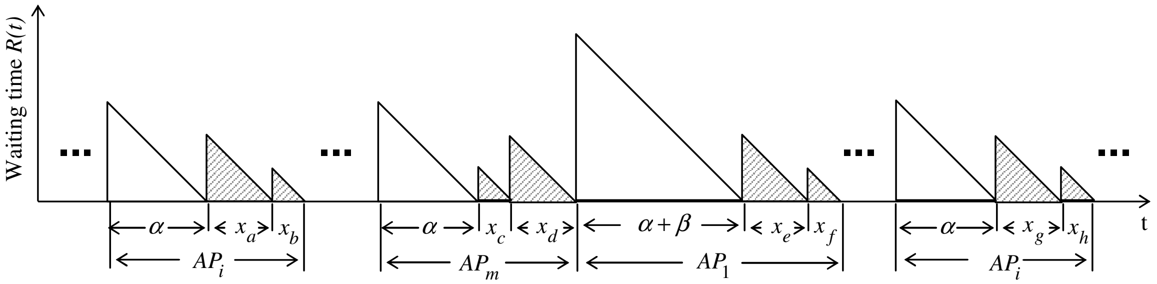


Fig. 2. Illustration of waiting time as function of packet arrival time t

where $\rho = \lambda\overline{X}$ is the average traffic intensity offered to the system. Combining (3) and (4), the expected value of packet ϵ's queuing delay is computed as:

$$E(D) = \frac{1}{1-\rho}\left[E(R) + E(G)\right] \tag{5}$$

Now the evaluation of average packet queuing delay is translated into finding the expected value of R and G. The former can be derived using a modified approach for the analysis of $M/G/1$ system with vacations in [8], as in our case there are multiple queues and also the vacation periods, i.e., the overhead portions in Fig. 1, are not allowed to appear continuously. Specifically, the value of R, as function of packet ϵ's arrival time t, i.e., $R(t)$, is illustrated in Fig. 2, where AP_i and x_k denote the arrival period of ONU i and the transmission time of the k^{th} packet serviced by the system, respectively. It is shown in the figure that the overhead portion (α or $\alpha+\beta$), i.e., vacation, alternates with the data transmission interval. Moreover, the overhead portion of the same ONU's AP appears one time in every m vacations. Suppose that at time t_0 the system has reached steady state and an arrival period of ONU m, i.e., AP_m, has just expired. Considering that triangles in Fig. 2 are all right-angled and isoceles, the average value of R for ONU i, denoted as $\overline{R_i}$, can be computed as:

$$\overline{R_i} = \frac{1}{w_i t_0}\left[\sum_{x_n \in S_i(t_0)} \frac{1}{2}x_n^2 + \frac{1}{2}v_i^2 L_i(t_0)\right] \tag{6}$$

where we denote w_i as the proportion of ONU i's APs occupying time interval $[0,\, t_0)$, v_i the length of vacation portion in ONU i's AP (α or $\alpha+\beta$), $L_i(t_0)$ the number of vacations appeared in ONU i's APs during time interval $[0,\, t_0)$, and $S_i(t_0)$ the set of packets serviced by ONU i during $[0,\, t_0)$, respectively. Since each ONU equally shares the offered traffic load and the packet size is independent from the arrival process, when t_0 increases, the number of packets serviced by ONU i during time interval $[0,\, t_0)$ can be more and more precisely estimated as $M(t_0)/m$, where $M(t_0)$ is the number of packets serviced by the system during time interval $[0,\, t_0)$. Moreover, the number of vacations appeared in ONU i's APs during $[0,\, t_0)$ is just $L(t_0)/m$, with $L(t_0)$ denoting the total number of vacations occurred during $[0,\, t_0)$, as these vacations appearing in ONU i's APs repeat once in every m consecutive vacations. Therefore, (6) can be represented as:

$$\begin{aligned}\overline{R_i} &= \frac{1}{w_i t_0}\left[\frac{1}{m}\sum_{k=1}^{M(t_0)} \frac{1}{2}x_k^2 + \frac{1}{2}v_i^2\frac{L(t_0)}{m}\right] \\ &= \frac{1}{2mw_i}\left[\frac{M(t_0)}{t_0}\frac{\sum\limits_{k=1}^{M(t_0)} x_k^2}{M(t_0)} + v_i^2\frac{L(t_0)}{t_0}\right]\end{aligned} \tag{7}$$

Noting that $\lim\limits_{t_0\to\infty}\dfrac{M(t_0)}{t_0} = \lambda$ and $\lim\limits_{t_0\to\infty} L(t_0) = t_0(1-\rho)/\overline{v}$, where $\overline{v} = \varphi/m$ denotes the average duration of one vacation period and ρ is also the average uplink utilization, we can infer that when $t_0 \to \infty$, the steady state value of $\overline{R_i}$ is given as:

$$\begin{aligned}\overline{R_i} &= \frac{1}{2mw_i}\left[\lambda\overline{x_k^2} + (1-\rho)\frac{v_i^2}{\overline{v}}\right] \\ &= \frac{1}{2w_i}\left[\frac{\lambda}{m}\overline{x_k^2} + (1-\rho)\frac{v_i^2}{\varphi}\right]\end{aligned} \tag{8}$$

The average value of R, i.e, $E(R)$, over the entire system then can be obtained as:

$$E(R) = \sum_{i=1}^{m} p_i(\epsilon)\overline{R_i} \tag{9}$$

where $p_i(\epsilon)$ denotes the probability for packet ϵ to arrive during an AP of ONU i. Noting that when $t_0 \to \infty$ the value of w_i approaches to $p_i(\epsilon)$, (8) and (9) indicate that:

$$E(R) = \frac{1}{2}\left[\lambda\overline{X^2} + \frac{(1-\rho)}{\varphi}\sum_{i=1}^{m} v_i^2\right] \tag{10}$$

where we use $\overline{X^2} = \overline{x_k^2}$ to concisely represent the second order moment of packet transmission time. Considering that $v_i = \alpha+\beta$, $(i = 1)$ for AP_1 and $v_i = \alpha$, $(2 \le i \le m)$ for other $m-1$ APs, the value of $E(R)$ ends up with:

$$\begin{aligned}E(R) &= \frac{1}{2}\left\{\lambda\overline{X^2} + \frac{(1-\rho)}{\varphi}\left[(m-1)\alpha^2 + (\alpha+\beta)^2\right]\right\} \\ &= \frac{1}{2}\left[\lambda\overline{X^2} + (1-\rho)\frac{m\alpha^2 + \beta^2 + 2\alpha\beta}{\varphi}\right]\end{aligned} \tag{11}$$

To find the value of $E(G)$ in (5), it is given by probability theory that the expected value of G can be obtained as:

$$E(G) = \sum_{i=1}^{m} \sum_{j=0}^{m-1} p_i^j(\epsilon) G_i^j(\epsilon) \tag{12}$$

where $G_i^j(\epsilon)$ is the value of G when packet ϵ arrives during ONU i's AP at its j^{th} FN, and $p_i^j(\epsilon)$ is the corresponding stationary probability when the system reaches steady state. Since the overhead portion in ONU 1's AP is longer than the one in other APs (see Fig. 1), the calculation of $E(G)$ is split into two parts, i.e.,

$$E(G) = \sum_{j=0}^{m-1} p_1^j(\epsilon) G_1^j(\epsilon) + \sum_{i=2}^{m} \sum_{j=0}^{m-1} p_i^j(\epsilon) G_i^j(\epsilon) \tag{13}$$

Considering that ρ is also the steady state link utilization, in Fig. 1 the time proportions occupied by data transmission intervals and overhead times are therefore ρ and $1-\rho$, respectively. Given the memoryless property of Poisson arrivals, this equivalently indicates that the steady state probability that packet ϵ arrives during any data transmission interval is ρ and during any overhead time period is $(1-\rho)$. Moreover, in steady state the length of data transmission interval in every AP shares the same average value, as ONUs are equally loaded. The probability that packet ϵ arrives during ONU i's AP, i.e, $p_i(\epsilon)$ is then given as:

$$\begin{aligned} p_1(\epsilon) &= \frac{1}{m}\rho + \frac{\alpha+\beta}{\varphi}(1-\rho) \\ p_i(\epsilon) &= \frac{1}{m}\rho + \frac{\alpha}{\varphi}(1-\rho) \qquad (2 \le i \le m) \end{aligned} \tag{14}$$

Since each ONU shares $1/m$ proportion of the overall average arrival rate λ, whenever packet ϵ arrives, it belongs to every ONU with probability of $1/m$. Mathematically, this implies that the conditional probability that packet ϵ arrives at ONU i's j^{th} FN, given that it arrives during ONU i's AP, i.e., $p^j(\epsilon|i)(1 \le i \le m,\ 0 \le j \le m-1)$, is $1/m$. Combining with (14), the probability of $p_i^j(\epsilon)$ in (13) results in:

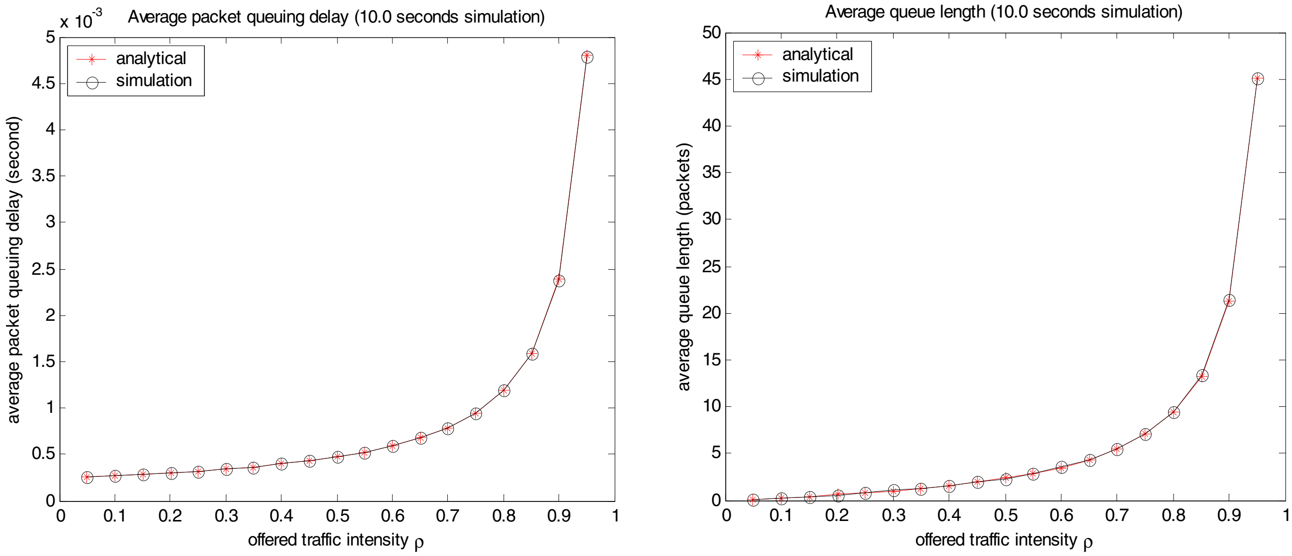
$$\begin{aligned} p_i^j(\epsilon) &= p^j(\epsilon|1)p_1(\epsilon) \\ &= \frac{1}{m}\left[\frac{1}{m}\rho + \frac{\alpha+\beta}{\varphi}(1-\rho)\right] \\ &\qquad\qquad (0 \le j \le m-1) \\ p_i^j(\epsilon) &= p^j(\epsilon|i)p_i(\epsilon) \\ &= \frac{1}{m}\left[\frac{1}{m}\rho + \frac{\alpha}{\varphi}(1-\rho)\right] \\ &\qquad\qquad (2 \le i \le m,\ 0 \le j \le m-1) \end{aligned} \tag{15}$$

The value of $G_i^j(\epsilon)$ in (13) is to be analyzed by two cases:

1) The j^{th} FN of ONU i is inclusively between ONU i and ONU m, i.e., $0 \le j \le m-i$. In this case after packet ϵ arrives, certain number of gap intervals will appear before its parent ONU is polled, whereby the transmission time of packet ϵ is requested. Namely, the gap interval immediately appears before the data transmission interval of ONU $i+1$, ONU $i+2$, until ONU $i+j$. Particularly, no gap interval will appear when $j = 0$. Therefore, the amount of overhead time consumed by these gap intervals is $j\alpha$. Note that any arriving packet has first to wait until its transmission time is requested by the nearest upcoming *REPORT* message of the parent ONU. Then in the following transmission opportunity of the parent ONU, this packet can be ultimately transmitted.
2) The j^{th} FN of ONU i is inclusively between ONU 1 and ONU $i-1$, i.e., $m-i+1 \le j \le m-1$ (see Fig. 1). In this case after packet ϵ arrives, certain number of gap intervals and a propagation interval will appear before its parent ONU is polled, whereby the transmission time of packet ϵ is requested. Namely, the gap interval immediately appears before the data transmission interval of ONU $i+1$, until ONU m, then ONU 1 until ONU $i+j-m$, and the propagation interval appears before ONU 1's data transmission interval. Therefore, the amount of time consumed by these gap intervals and the propagation interval is $j\alpha + \beta$.

In both cases above, once the transmission time of packet ϵ is requested, an extra set of gap intervals and a propagation interval are required before the transmission of packet ϵ starts. This amount of time consumption is just φ, the total overhead

$$\begin{aligned} E(G) &= \sum_{j=0}^{m-1} p_i^j(\epsilon) G_1 j(\epsilon) + \sum_{i=2}^{m}\left[\sum_{j=0}^{m-i} p_i j(\epsilon) G_i^j(\epsilon) + \sum_{j=m-i+1}^{m-1} p_i^j(\epsilon) G_i^j(\epsilon)\right] \\ &= \sum_{j=0}^{m-1} \frac{1}{m}\left[\frac{1}{m}\rho + \frac{\alpha+\beta}{\varphi}(1-\rho)\right](j\alpha+\varphi) \\ &\quad + \sum_{i=2}^{m}\left\{\sum_{j=0}^{m-i} \frac{1}{m}\left[\frac{1}{m}\rho + \frac{\alpha}{\varphi}(1-\rho)\right](j\alpha+\varphi) + \sum_{j=m-i+1}^{m-1} \frac{1}{m}\left[\frac{1}{m}\rho + \frac{\alpha}{\varphi}(1-\rho)\right](j\alpha+\beta+\varphi)\right\} \end{aligned} \tag{17}$$



(a) Average packet queuing delay by 10.0 seconds simulation.

(b) Average queue length per ONU by 10.0 seconds simulation.

Fig. 3. Performance with Poisson traffic inputs

time occurred in one service cycle. Therefore, the value of $G_i^j(\epsilon)$ in (13) is finally computed as:

$$\begin{aligned} G_i^j(\epsilon) &= j\alpha + \varphi \qquad (1 \le i \le m,\, 0 \le j \le m-i) \\ G_i^j(\epsilon) &= j\alpha + \beta + \varphi \\ &\qquad (1 \le i \le m,\, m-i+1 \le j \le m-1) \end{aligned} \tag{16}$$

Collecting (15) and (16), (13) yields (17), which is given on the last page. Substituting (11) and (17) into (5), with some simplifications, the steady state average packet queuing delay ends up with:

$$E(D) = \frac{1}{2(1-\rho)}\left[\lambda\overline{X^2} + 3\varphi - \frac{\varphi}{m}\rho\right] \tag{18}$$

B. Average queue size

With the average packet queuing delay derived in (18), the steady state average queue length, i.e., average number of packets staying in the queue of one ONU, denoted as $E(N)$, is given by Little's theorem as [8]:

$$\begin{aligned} E(N) &= \frac{\lambda}{m}E(D) \\ &= \frac{\lambda}{2m(1-\rho)}\left[\lambda\overline{X^2} + 3\varphi - \frac{\varphi}{m}\rho\right] \end{aligned} \tag{19}$$

IV. SIMULATION EXPERIMENTS

We developed a simulative EPON network over the platform of *ns-2* [9], to verify the above analysis. The relevant parameters used for the simulation are listed in Table I. We have neglected the OLT computation time and ONU message processing time, as they do not qualitatively affect our analysis.

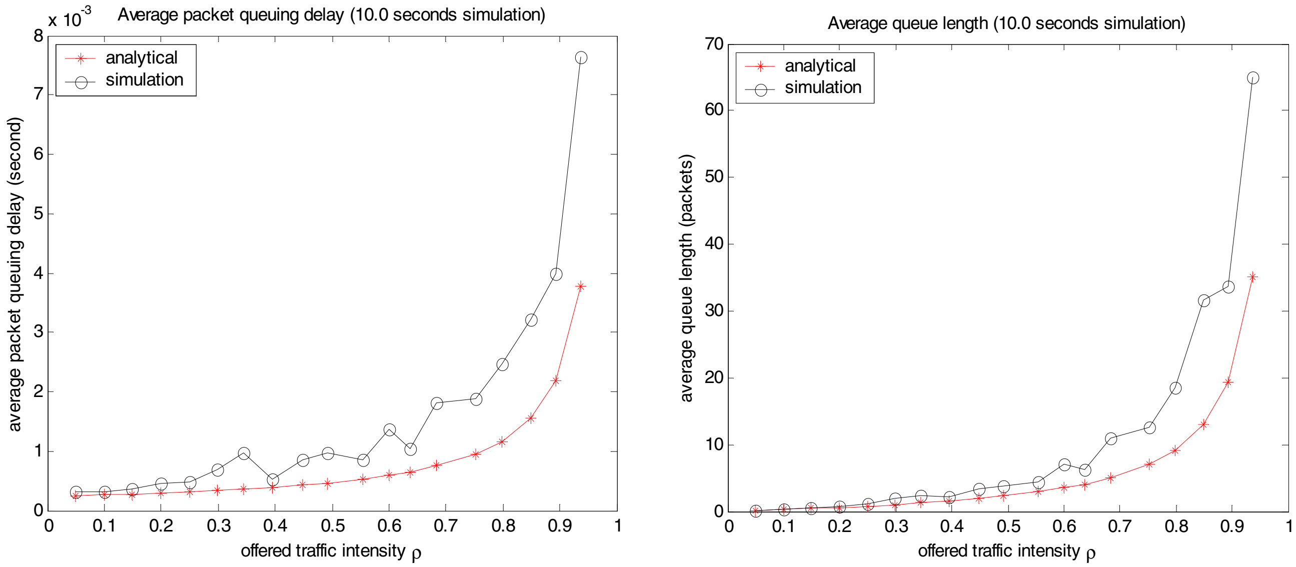
In the simulation, we sampled the number of queued packets, i.e., queue length, at a tagged ONU every $2ms$ to compare with the value obtained through theoretical derivation.

TABLE I
SIMULATION PARAMETERS

number of OLT	1
number of ONU	16
uplink capacity	1 *Gbps*
OLT-ONU distance	20 km
guard time	1 μs
REPORT message size	64 *bytes*
GATE message size	64 *bytes*
data packet size	uniformly $\in$ [64, 1518] *bytes*
OLT computation time	*neglected*
ONU message processing time	*neglected*

Fig. 3(a) and Fig. 3(b) illustrate the average packet queuing delay and average queue length, respectively, obtained by simulation and theoretical analysis. We can see the perfect match of the simulated curve with corresponding theoretical calculation. One should notice that, however, the perfect match between simulation and analysis depends on, for different loading scenarios, enough time to permit the queue in each ONU be built-up thereby the steady state queuing process is achieved. For example, if the observation is based only on 0.1 second simulation run, we have recognized good match of these two curves only up to around $\rho = 0.7$.

The correctness of the analytical model discussed before largely depends on the memoryless assumption of the offered traffic pattern. It is well known that traffic passing through access networks is bursty and possesses *long range dependency* (LRD) [10]. Without claiming theoretical certainty, we also compared the derived analytical framework with the performance obtained by self-similar traffic inputs, to visualize the confidence interval for applying our analysis over real network scenario. Table II includes the values of *Hurst parameter* ($\mathcal{H}$)



(a) Average packet queuing delay by 10.0 seconds simulation.

(b) Average queue length per ONU by 10.0 seconds simulation.

Fig. 4. Performance with self-similar traffic inputs

measured in the simulation[3]. Fig. 4(a) and Fig. 4(b) illustrate the difference between simulation outcome and our analysis, in terms of average packet queuing delay and average queue length. We can see that for light loading scenarios, our analysis for memoryless traffic profiles is also able to estimate the network performance for bursty traffic inputs ($\mathcal{H} < 0.7$), with small deviation. For example, when the network is loaded with less than 60% of its uplink capacity, the deviation can be limited within 40%.

TABLE II
MEASURED HURST PARAMETERS IN SIMULATION

$(\rho,\ \mathcal{H})$	$(0.0498,\ 0.7367)$	$(0.1012,\ 0.7325)$
$(0.1495,\ 0.7450)$	$(0.1988,\ 0.7518)$	$(0.2506,\ 0.7623)$
$(0.2976,\ 0.7832)$	$(0.3445,\ 0.7871)$	$(0.3942,\ 0.7440)$
$(0.4475,\ 0.7715)$	$(0.4917,\ 0.7708)$	$(0.5541,\ 0.7592)$
$(0.6002,\ 0.7693)$	$(0.6360,\ 0.7532)$	$(0.6841,\ 0.7725)$
$(0.7529,\ 0.7830)$	$(0.7976,\ 0.7764)$	$(0.8485,\ 0.7661)$
$(0.8920,\ 0.7567)$	$(0.9375,\ 0.7601)$	

V. CONCLUSION

In this study, we analyzed the performance of an EPON system applying quasi-leaved polling operation and gated service scheme for resource allocation. A graphical representation was formed to investigate the temporal behavior of such a system. We also derived close-form expressions to evaluate the average packet queuing delay and average queue length. Simulation verified that this analytical model can precisely evaluate the network performance for memoryless traffic input, and is capable of estimating the system performance with small deviation, for bursty traffic profiles.

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[3] For each value of ρ, the value of $\mathcal{H}$ shown in the table is the averaged value of Hurst parameters for 16 traffic traces feeding the network. The Hurst parameter of each traffic trace is estimated by *least square approximation* and the measurement scale varies from $0.0625ms$ to $1ms$.