A NEW MEASUREMENT TECHNIQUE FOR TRACKING VOLTAGE PHASORS, LOCAL SYSTEM FREQUENCY, AND RATE OF CHANGE OF FREQUENCY

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Abstract: With the advent of Substation Computer Systems dedicated to protection, control and data logging functions in a Substation, it becomes possible to develop new applications which can utilize the processing power available within the substation. The microcomputer based Symmetrical Component Distance Relay (SCDR) described in the references cited at the end of this paper possesses certain characteristics which facilitate real-time monitoring of positive sequence voltage phasor at the local power system bus. With a regression analysis the frequency and rate-of-change of frequency at the bus can also be determined from the positive sequence voltage phase angle. This paper describes the theoretical basis of these computations and describes results of experiments performed in the AEP power system simulation laboratory. Plans for future field tests on the AEP system are also outlined.

INTRODUCTION

Positive sequence voltage phasor at a power system bus is a parameter of vital significance. The collection of all positive sequence voltage phasors constitutes the state-vector of a power system. It is also one of the output variables of such steady-state analysis programs as the load flow and state-estimation. The idea of measuring these voltage phasors directly has appeared in the technical literature from time to time. However, no entirely satisfactory practical method for measuring voltage phasors in real-time has yet been proposed.

It is reasonable to assume that a measurement system capable of measuring positive sequence voltage phasors directly and with adequate accuracy has the potential to simplify and improve state-estimation algorithms as well as other real-time analysis programs. In addition, accurate measurement of phasor voltages in real-time may lead to development and implementation of adaptive control procedures which can be based on direct measurement of key system variables. Use of local frequency and rate of change of frequency for load-shedding and load restoration functions is yet another well-known application of such measurements. In fact, the measurement of phase angle, frequency and rate of change of frequency (which are quantities of "system" significance) with substation computers can be viewed as a building block for the computer based distributed-processor, hierarchical control systems which have been proposed for real-time monitoring and control of power systems.

The steady-state analysis programs such as load-flow, state-estimation and optimal power flow use the positive sequence bus voltage as the state variable, and the network model uses the positive sequence representation of the power system. It should therefore be a matter of some concern that many of the phase angle measurement schemes proposed in the literature [1] measure the magnitude and phase angle of voltage of one phase. In the presence of system unbalance (which exists on almost all high voltage systems) such a measurement is an approximation at best and does not represent the positive sequence voltage at the bus accurately. Furthermore, almost all of these schemes measure the phase angle of the voltage by timing the zero-crossing instant of the voltage waveform with respect to a standard reference pulse. Since the zero-crossing instant of a voltage wave is affected by the magnitude of the non - 60 Hz components present, this is yet another source of error as far as the measurement of positive sequence fundamental frequency voltage measurement is concerned. In contrast, the technique described in this paper measures a digitally filtered positive sequence voltage phasor at the bus directly.

Recently, papers [2,9] have described techniques for measuring the frequency and rate-of-change of frequency with a microcomputer. This technique uses the aggregate magnitude of all non - 60 Hz components in a waveform as a measure of the input waveform frequency. It has been observed that this measurement principle (based on "leakage effects" in the Fourier Transform calculation) is not a particularly sensitive measure of frequency deviations, and may in fact go completely astray if a non - 60 Hz component is present in the input waveform. In contrast, this paper describes a robust computation technique for phase angle, frequency, and rate-of-change of frequency.

Following a theoretical derivation of the necessary equations, a laboratory experiment using a small scale power system model will be described and the results of the experiment discussed. Plans for field tests of these ideas on the AEP system will also be presented.

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FILTERED PHASORS FROM SAMPLED DATA

Consider a sinusoidal input signal of frequency ω given by

$$x(t) = \sqrt{2}X \sin(\omega t+\phi) \tag{1}$$

This signal is conventionally represented by a phasor (a complex number) $\bar{X}$

$$\bar{X} = Xe^{j\phi} = X\cos\phi + jX\sin\phi \tag{2}$$

Assuming that x(t) is sampled N times per cycle of the 60 Hz waveform to produce the sample set $\{x_k\}$

$$x_k = \sqrt{2}\, X \sin\left(\frac{2\pi}{N}k+\phi\right) \tag{3}$$

The Discrete Fourier Transform of $\{x_k\}$ contains a fundamental frequency component given by

$$\bar{X}_1 = \frac{2}{N}\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}k}$$
$$= \frac{2}{N}\sum_{k=0}^{N-1} x_k \cos\frac{2\pi}{N}k - j\frac{2}{N}\sum_{k=0}^{N-1} x_k \sin\frac{2\pi}{N}k \tag{4}$$

$$\equiv X_c - jX_s \tag{5}$$

where X_c and X_s are the cosine and sine multiplied sums in the expression for $\bar{X}_1$ Substituting for x_k from equation (3) in equations (4) and (5) it can be shown that for a sinusoidal input signal given by equation (1),

$$X_c = \sqrt{2}\, X \sin\phi$$
$$X_s = \sqrt{2}\, X \cos\phi \tag{6}$$

From equations (2), (5) and (6), it follows that the conventional phasor representation of a sinusoidal signal is related to the fundamental frequency component of its DFT by

$$\bar{X} = \frac{1}{\sqrt{2}}\, j\, \bar{X}_1 = \frac{1}{\sqrt{2}}(X_s + j\, X_c) \tag{7}$$

In the preceding development it was assumed that the input signal is a pure sine wave of fundamental frequency. When the input contains other frequency components as well, the phasor calculated by equation (7) is a filtered fundamental frequency phasor. The input signals must be band-limited to satisfy the Nyquist criterion[3] to avoid errors due to aliasing effects. It is therefore assumed that the input signals are filtered with low-pass analog filters having a cut-off frequency of $\omega N/4\pi$ Hz. The effect of anti-aliasing analog filters on the fundamental frequency signals has been discussed in reference [4].

Another point to note is that equation (4) assumes data collected over one complete cycle of the fundamental frequency. Although the filter equations (4) are particularly simple for the case of a one cycle data window, similar filter equations can be formulated for any other window length. [5] The consequence of using other window lengths is to affect the accuracy of the phasor computation. A more detailed discussion of this aspect will be found in reference [5]. The remainder of this paper will assume the use of one cycle data window, it being a simple matter to modify one cycle data-window results to reflect the effects of other data window lengths.

RECURSIVE PHASOR COMPUTATION

The input signal of equation (1) is shown in Figure (1). Data window 1 produces the sample set $\{x_k, k=0,\dots N-1\}$ and the phasor representation obtained from this sample set is given by equation (7). A new sample is obtained after an elapsed time corresponding to the sampling angle $2\pi/N$ radians. At this time the data window 2 becomes operative with sample set $\{x_k, k=1,\dots\dots N\}$. The phasor computations using data window 2 are performed with equations (1), (2) and (7) as follows:

$$x(t) = \sqrt{2}X \sin\left(\omega t+\phi+\frac{2\pi}{N}\right) \tag{8}$$

$$\bar{X}^{(new)} = X\, e^{j\left(\phi+\frac{2\pi}{N}\right)}$$
$$= \bar{X}^{(old)}\, e^{j\frac{2\pi}{N}} \tag{9}$$

$$\bar{X}^{(new)} = \frac{1}{\sqrt{2}}\, j\, \bar{X}_1^{(new)}$$
$$= \frac{1}{\sqrt{2}}\left(X_s^{(new)} + j\, X_c^{(new)}\right) \tag{10}$$

where the superscripts 'new' and 'old' signify computations from data windows 2 and 1 respectively. Equation (9) shows that the use of equation (7) for calculating the filtered phasor of an input signal produces a phasor which rotates in a counterclockwise direction in the complex

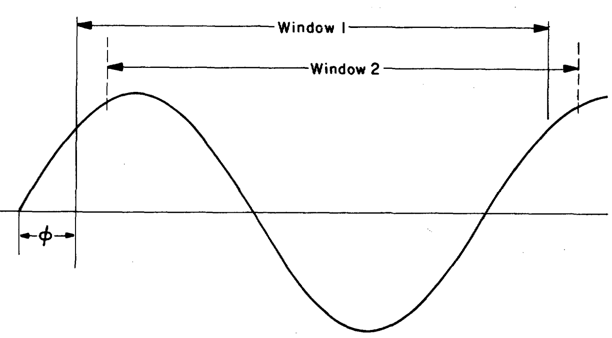

figure 1
Data Windows

plane by the sampling angle $2\pi/N$. This phenomenon is illustrated in Figure 2. The angular velocity of the phasor computed from a 60 Hz input signal is thus $120\pi \simeq 377$ radians per second, although the phasor is available only at discrete angles.

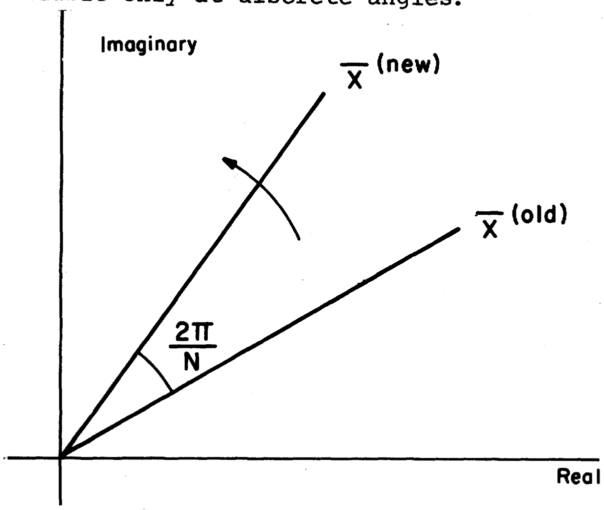

figure 2
Phasors from Different Data Windows

In general, when sample data from the r^{th} window is used,

$$X_c^{(r)} = \frac{2}{N} \sum_{k=0}^{N-1} x_{k+r-1} \cos \frac{2\pi}{N} k \qquad (11)$$

$$X_s^{(r)} = \frac{2}{N} \sum_{k=0}^{N-1} x_{k+r-1} \sin \frac{2\pi}{N} k \qquad (12)$$

$$\bar{X}^{(r)} = \frac{1}{\sqrt{2}} (X_s^{(r)} + j X_c^{(r)})$$

$$= \bar{X}^{(r-1)} \cdot e^{j \frac{2\pi}{N}} \qquad (13)$$

Clearly the procedure described by (11)-(13) is non-recursive, and requires 2N multiplications and 2(N-1) additions to produce the phasor $\bar{X}^{(r)}$. (The factor 2 is of no consequence, and is usually suppressed). It should however be noted that in progressing from one data window to the next, only one sample (x_o) is discarded and only one sample (x_N) is added to the data set. It is therefore advantageous to develop a technique which retains 2(N-1) multiplications and 2(N-1) sums corresponding to that portion of the data which is common to the old and new data windows.

A recursive computation of the type described above is made possible by the fact that the DFT computation is arbitrary to the extent of its phase angle. Consider the calculation of $X_c^{(\theta)}$, $X_s^{(\theta)}$ and $\bar{X}^{(\theta)}$ with Fourier coefficients having an arbitrary phase angle θ:

$$X_c^{(\theta)} = \frac{2}{N} \sum_{k=0}^{N-1} x_k \cos (\frac{2\pi}{N} k + \theta) \qquad (14)$$

$$X_s^{(\theta)} = \frac{2}{N} \sum_{k=0}^{N-1} x_k \sin (\frac{2\pi}{N} k + \theta) \qquad (15)$$

$$\bar{X}^{(\theta)} = \bar{X} e^{-j\theta} \qquad (16)$$

The phasor representation of the input signal in equation (16) contains as much information as does the one described by equations (2) and (7), and can therefore be used without any loss of generality. It is advantageous to calculate the phasor for data window 1 with equations (2) and (7), and that for data window 2 with equations (14) - (16):

$$X_c^{(new),\theta} = \frac{2}{N} \sum_{k=0}^{N-1} x_{k+1} \cos (\frac{2\pi}{N} k + \theta) \qquad (17)$$

$$X_s^{(new),\theta} = \frac{2}{N} \sum_{k=0}^{N-1} x_{k+1} \sin (\frac{2\pi}{N} k + \theta) \qquad (18)$$

$$\bar{X}^{(new),\theta} = \bar{X}^{(old)} \cdot e^{-j\theta} \qquad (19)$$

If θ is now made equal to $\frac{2\pi}{N}$, equations (17) and (18) become

$$X_c^{(new),2\pi/N} = \frac{2}{N} \sum_{k=o}^{N-1} x_{k+1} \cos \frac{2\pi}{N}(k+1) \qquad (20)$$

$$X_s^{(new),2\pi/N} = \frac{2}{N} \sum_{k=o}^{N-1} x_{k+1} \sin \frac{2\pi}{N} (k+1) \qquad (21)$$

Equations (20) and (21) are recursion relations, since they can be re-written as

$$X_c^{(new),\frac{2\pi}{N}} = \frac{2}{N} \sum_{k=0}^{N-1} x_k \cos \frac{2\pi}{N} k$$

$$+ \frac{2}{N} \cos 2\pi (x_N - x_o) \qquad (22)$$

$$X_s^{(new),\frac{2\pi}{N}} = \frac{2}{N} \sum_{k=0}^{N-1} x_k \sin \frac{2\pi}{N} k$$

$$+ \frac{2}{N} \sin 2\pi (x_N - x_o) \qquad (23)$$

or

$$X_c^{(\text{new}),\ \frac{2\pi}{N}} = X_c^{(\text{old})} + \frac{2}{N}\cos 2\pi(x_N - x_o) \quad (24)$$

$$X_s^{(\text{new}),\ \frac{2\pi}{N}} = X_s^{(\text{old})} + \frac{2}{N}\sin 2\pi(x_N - x_o) \quad (25)$$

If it is understood that the angle θ is always set equal to $2\pi/N$, this factor can be dropped from the superscript on the left hand side of equations (24) and (25), and the new phasor is given by

$$\bar{X}^{(\text{new})} = \bar{X}^{(\text{old})} + \frac{1}{j\sqrt{2}}\cdot\frac{2}{N}\,(x_N - x_o)\,e^{-j\frac{2\pi}{N}} \quad (26)$$

In general, the r^{th} phasor is computed from the (r-1)th phasor by

$$\bar{X}^{(r)} = \bar{X}^{(r-1)} + \frac{1}{j\sqrt{2}}\,\frac{2}{N}\,(x_{N+r} - x_r)\,e^{-j\frac{2\pi}{N}(r-1)} \quad (27)$$

Recursive equations (24) and (25) are comparable to the non-recursive equations (11) and (12). With the recursive procedure only two multiplications need be performed at each new sample time; making this a very efficient computational algorithm.

It is interesting to note that when the input signal is a pure sine wave of fundamental frequency $x_{N+r} = x_r$ for all r; and consequently for this case equation (27) becomes

$$\bar{X}^{(r)} = \bar{X}^{(r-1)} \quad \text{for all r.} \quad (28)$$

Equation (28) shows that when a recursive computation is used to calculate phasors, it leads to stationary phasors in the complex plane when the input signal is a pure sine wave of fundamental frequency. Recall that non-recursive phasor computation leads to phasors which rotate in the complex plane with an angular velocity ω.

RECURSIVE SYMMETRICAL COMPONENT COMPUTATION

Recursive computation of symmetrical components of three phase signals has been described in literature [4,6,7]. A few salient points of that procedure are summarized here for ready reference. A sampling rate of 720 Hz (N=12) has been found to be most beneficial for real-time computation of symmetrical components [4]. The sines and cosines of multiples of 30° ($2\pi k/N$) are shown arranged around the circumference of a circle in Figure 3. Computation of X_c and X_s of equation (5) can be visualized as an inner product of the data vector $\{x_k,\ k=o\ldots.\ N-1\}$ and the constants of Figure 3 starting at constant no. 1 and constant no. 4 respectively.

It will be recalled that the positive sequence voltage can be calculated from three phase voltages by the relation

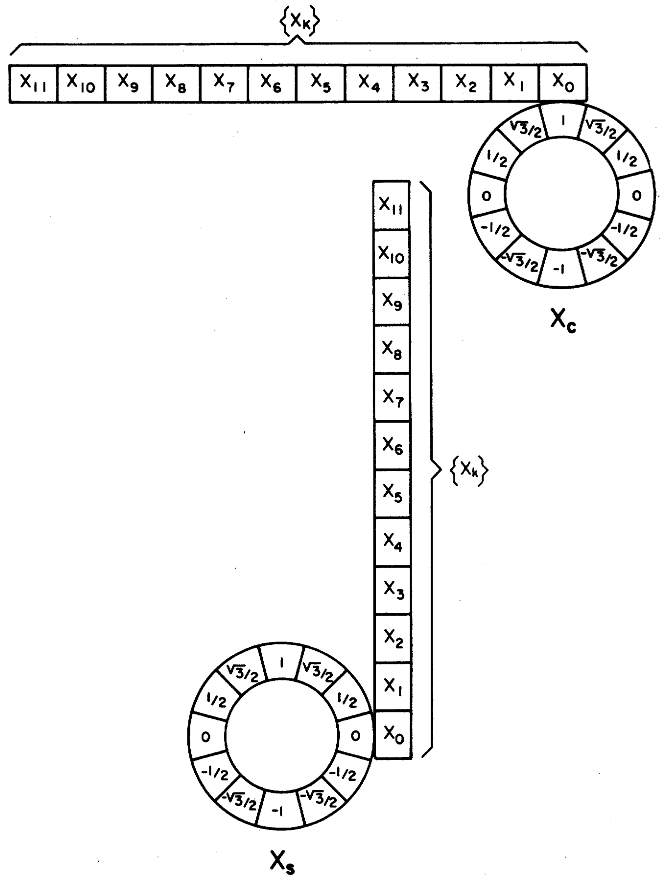

figure 3
Phasor from Samples

$$\bar{V}_1 = \frac{1}{3}\,(\bar{V}_a + \alpha\bar{V}_b + \alpha^2\bar{V}_c) \quad (29)$$

where α and α^2 are phase shifts of 120° and 240° respectively. Equations (14) and (15) provide the necessary phase shifts for phase b and phase c voltages. Since the sampling rate selected (720 Hz) corresponds to a phase angle of 30° between samples, the necessary phase shifts are obtained simply by introducing an offset of 4 and 8 respectively in using the constant table of Figure 3. Thus $\alpha\,\bar{V}_b$ is produced by calculating X_c and X_s starting with constants no. 5 and 9 on data samples of $v_b(t)$, while $\alpha^2\,\bar{V}_c$ is obtained when X_c and X_s are calculated starting with constants no. 9 and 12 on data samples of $v_c(t)$. The recursive relation for the positive sequence voltage is similar to equation (27)

$$\bar{V}_1^{(\text{new})} = \bar{V}_1^{(\text{old})} + j\frac{2}{N}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{3}\Big[(v_{a,r+N} - v_{a,r})\,e^{-j\frac{2\pi}{N}(r-1)} + (v_{b,r+N} - v_{b,r})\,e^{-j\frac{2\pi}{N}(r+5-1)} + (v_{c,r+N} - v_{c,r})\,e^{-j\frac{2\pi}{N}(r+9-1)}\Big] \quad (30)$$

Other symmetrical components of voltages and currents are calculated in a similar manner. Several practical considerations for computing symmetrical components will be found in references [4,6,7]. It should also be reiterated that although equation (30) assumes data window of one cycle, other data windows can be readily accommodated in this procedure.

It will be assumed that a positive sequence voltage phasor is computed recursively at each sampling instant (12 times a cycle). This computation is done by the Symmetrical Component Distance Relay as part of its relaying algorithm, and if such a relay exists at the substation this phasor is available for use at no additional computational burden. In the absence of a Symmetrical Component Distance Relay at the substation, equation (30) must be programmed on a microprocessor and installed at the substation as a stand-alone measurement system.

CALCULATION OF LOCAL FREQUENCY

Assume that the sampling clock used for obtaining the sampled data from the three phase voltage inputs operates precisely at 720 Hz while the power system frequency is precisely 60 Hz. When a recursive relation such as equation (27) is used to calculate the phasors, the resultant phasors will remain stationary in the complex plane. The equations derived in this section will consider phasor computations from a single input signal, although it may be verified readily that the results of this section apply directly to the positive sequence voltage calculated from three input voltage signals according to equation (30).

If the input signal frequency is now assumed to change slightly from 60 Hz by an amount Δf, while the sampling clock frequency remains at 720 Hz, it can be shown that the recursive relation of equation (27) changes into

$$\bar{X}^{(r)}_{60+\Delta f} = \bar{X}^{(o)}_{60} \cdot \frac{\sin \frac{\Delta f}{60}\pi}{N \sin \frac{\Delta f}{60}\frac{\pi}{N}} \cdot e^{j \frac{\Delta f}{60} \frac{2\pi}{N} r} \tag{31}$$

where $\bar{X}^{(o)}_{60}$ is the initial computation of the phasor from a 60 Hz input signal having the same magnitude as the $(60+\Delta f)$ Hz signal, r is the recursion number, and N is the number of samples in a period of the 60 Hz wave. Equation (31) shows that when the input signal frequency changes from 60 Hz to $(60 + \Delta f)$ Hz, the phasor obtained recursively undergoes two modifications:

a magnitude factor of $(\sin \frac{\Delta f}{60}\pi / N \sin \frac{\Delta f}{60} \frac{\pi}{N})$;

and a phase factor of $\exp (j\frac{\Delta f}{60} \frac{2\pi}{N} r)$. The magnitude factor is independent of r, and is relatively small for small changes in frequency. The magnitude factor is a manifestation of the "leakage effect", and has been proposed as a measure of the frequency deviation Δf [2].

However, the phase angle effect is far more sensitive to the frequency Δf, and provides a most direct measure of Δf.

Denoting the phase factor by $\exp (j\psi_r)$

$$e^{j(\frac{\Delta f}{60} \frac{2\pi}{N} r)} \equiv e^{j\psi_r} \tag{32}$$

$$\psi_r = \frac{\Delta f}{60} \frac{2\pi}{N} r \tag{33}$$

and thus the phase angle at rth recursive computation directly depends upon the frequency deviation and the recursion order r. Since r increases by 1 in each iteration, the recursive relation for ψ_r becomes

$$\psi_r = \psi_{r-1} + \frac{\Delta f}{60} \cdot \frac{2\pi}{N} \tag{34}$$

Further, the time interval between two iterations is 1/60N seconds: and therefore the angular velocity of ψ is given by

$$\begin{aligned} \frac{d\psi}{dt} &= \frac{\psi_r - \psi_{r-1}}{(1/60N)} \\ &= 2\pi \, \Delta f \text{ radians/second} \end{aligned} \tag{35}$$

The rate of change of the complex phasor angle is thus directly related to the input signal frequency. For example, an input signal with frequency (60 ± 1) Hz would produce a phasor that turns one complete circle per second in the complex plane. When the input signal frequency is 61 Hz, the phasor rotates in the counterclockwise direction, whereas for an input signal frequency of 59 Hz the phasor rotates in a clockwise direction. There is a striking resemblance between this phenomenon of a rotating phasor and the principle of a power system synchroscope so familiar to most power system engineers. Just as the frequency is calculated by calculating $d\psi/dt$, the rate of change of frequency can be calculated by computing $d^2\psi/dt^2$. From equation (35)

$$\begin{aligned} f &= 60 + \Delta f \\ &= 60 + \frac{1}{2\pi} \frac{d\psi}{dt} \quad \text{Hz} \end{aligned} \tag{36}$$

$$\frac{df}{dt} = \frac{1}{2\pi} \frac{d^2\psi}{dt^2} \quad \text{Hz/second} \tag{37}$$

The actual computations for f and df/dt are performed with the help of regression formulas as explained below.

IMPLEMENTATION ON A MICROCOMPUTER

It is well known that a frequency meter (or a frequency measurement technique) has a longer time constant (i.e. takes longer to make the measurement) for input signals with low frequencies. It can therefore be expected that when input signals have frequencies which differ from 60 Hz by small amounts, the technique presented in this paper would take longer to make that measurement. This effect is linked to the accuracy requirement of the frequency measurement, and is also present in the algorithm developed to implement equations (36) and (37).

The real-time computation of phase angle, frequency and rate-of-change of frequency on a microcomputer requires careful consideration of the numerical technique to be selected. The first step in the algorithm is to calculate the phase angle between two phasors. Consider the two phasors OA and OB in Figure 4. The phase angle ψ between OA and OB is given by

$$\tan\frac{\psi}{2} = \frac{|\frac{1}{2}(OB-OA)|}{|\frac{1}{2}(OB+OA)|} \tag{38}$$

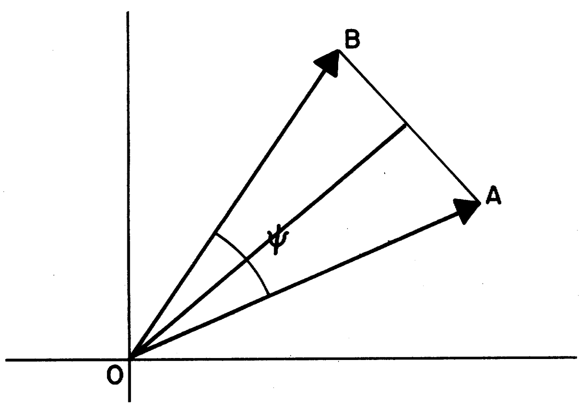

figure 4
Angle between two Phasors

The ratio on the right hand side of equation (38) is calculated first, and then ψ obtained from a stored table in the microcomputer. The ratio calculation can be simplified greatly by noting that (OB-OA) is perpendicular to (OB+OA); and for two phasors $\bar{X}$ and $\bar{Y}$ which are perpendicular to each other,

$$\frac{|\bar{X}|}{|\bar{Y}|} = \frac{|X_r + jX_i|}{|Y_r + jY_i|}$$

$$= \frac{|X_r| + |X_i|}{|Y_r| + |Y_i|} \tag{39}$$

Equation (39) is far simpler than equation (38), and is used in the on-line program to calculate ψ.

The denominator of equation (38) is the magnitude of positive sequence voltage at the substation. During non-fault periods, this magnitude is relatively unchanging. The Symmetrical Component Distance Relay developed at AEP uses internal scaling factors so as to produce a normal positive sequence voltage magnitude of about 25,000. It is well to design an algorithm such that the numerator in equation (38) turns out to be about 6000 in all cases so that the effect of truncation error in the numerator is negligible (less than 0.02%). A numerator of 6000 (with a denominator of 25,000) corresponds to $\psi/2$ of about 0.25 radian; or ψ, the angle between the two phasors of about 0.5 radian. Recall that the phasor computed from an input signal of frequency $(f+\Delta f)$ Hz rotates at an angular velocity of $2\pi\Delta f$ in the complex plane. Thus to generate a phase angle of 0.5 radians between two phasors, an input signal of $(f+\Delta f)$ Hz frequency must rotate for a time T given by

$$T = \frac{0.5}{2\pi\Delta f} \simeq \frac{0.08}{\Delta f} \text{ seconds} \tag{40}$$

Assuming that at least 4 raw phase angle measurements would be used to produce one smoothed frequency result (for example, by a simple linear regression formula), the time required to obtain the frequency reading of $(60 + \Delta f)$ Hz would be

$$T_{(60+\Delta f)} = 4T = \frac{0.32}{\Delta f} \text{ seconds} \tag{41}$$

A frequency of 60.1 Hz would thus be measured in 3.2 seconds, while a frequency of 65 Hz would be measured in 0.064 second. Assuming that $\pm$ 5 Hz is the limit of frequency deviation that may have to be measured, four phasor computations must be made available in about 0.064 seconds. A phasor computation frequency of about one computation per cycle is thus sufficient to measure frequencies from 55 Hz to 65 Hz with inaccuracies of numerical computation less than 0.02%.

Equation (41) shows the relationship between the frequency and its measurement time. It is clear that smaller Δf would require longer times to make the measurement. Notice however that this particular relationship is based upon specifying that the numerator of equation (38) be of the order of 6000 in order to limit the errors of computation to less than 0.02%. If greater errors are permissible (and in many application this may well be the case) the measurement time can be shortened in an inverse proportion. Thus if it takes 3.2 seconds to measure 60.1 Hz for a given error limit, it would take 1.6 seconds to measure the same frequency if the allowed error is doubled. Based upon arguments of this nature, an adaptive algorithm could be designed which measures frequencies to differing accuracy specification in different times.

It should also be noted that errors discussed here are those of computation alone; other sources of error - such as the effects of temperature variation on electronic component values etc. - have not been considered. In a complete system, all these errors must be accounted for in specifying an overall accuracy capability of the proposed measurement scheme.

Figure 5 is a flow chart of an algorithm which would measure frequencies in the range of 55 Hz to 66 Hz in measurement times as per equation (41).

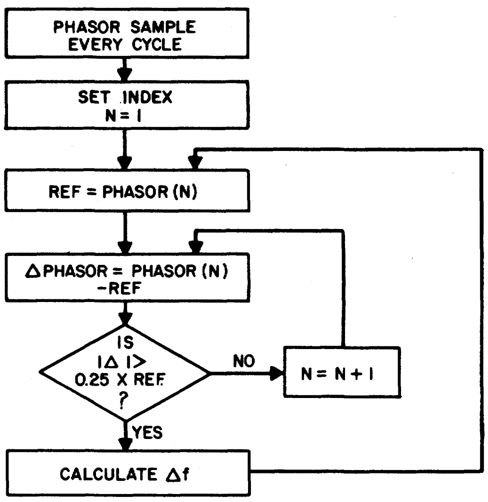

figure 5
Frequency Measurement Flowchart

EXPERIMENTAL RESULTS

The experiments described here were performed in the AEP power system simulation laboratory [8]. In the first series of experiments, the positive sequence voltage at a system bus having a nominal supply frequency of 60 Hz (local utility supply) was monitored by the Symmetrical Component Distance Relay. (See Figure 6)

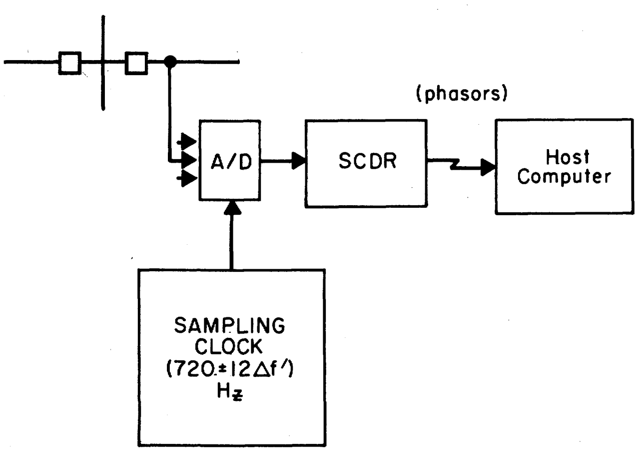

figure 6
Positive Sequence Voltage Measurement

The three phase voltage input was sampled at a variable clock frequency of $(720 + 12\Delta f')$ Hz, and the calculated positive sequence voltage phasors were stored in real time once every 12 samples in a table within the SCDR. These were then moved to the host computer where the algorithm of Figure 5 was executed in an off-line mode. Integer arithmetic was used in all key operations. The power supply frequency remained at a nominal frequency of 60 Hz, and differed from the fundamental frequency of the sampling clock by $\Delta f'$. Hz. As this experiment was meant to simulate a variable power system frequency and a fixed clock frequency of 720 Hz, the frequency difference $\Delta f'$ is related to the frequency difference Δf being simulated by the relation

$$\Delta f = \Delta f' \frac{60}{60+\Delta f'} \tag{42}$$

Table I shows the sampling clock setting converted to Δf according to equation (42), the measured Δf according to the algorithm of Figure 5, the average time to make that measurement, and the error of the measurement.

Note that the measurement time for a given Δf is approximately inversely proportional to Δf in keeping with the result of equation (41). The Δf in Table I could not be adjusted to a simple desired setting precisely because the sampling clock could only be adjusted to the nearest tenth of a microsecond.

TABLE I

Case No.	Δf Setting	Δf Measured	Error	Average Measuremt Time (Sec)
1	-5.002	-5.014	.012	0.097
2	-4.501	-4.495	-.006	0.089
3	-3.999	-3.994	-.005	0.088
4	-3.499	-3.490	-.009	0.133
5	-3.002	-2.995	-.007	0.133
6	-2.501	-2.491	-.01	0.136
7	-1.999	-1.990	-.009	0.167
8	-1.002	-0.992	-.01	0.308
9	-0.751	-0.736	-.015	0.421
10	-0.501	-0.489	-.012	0.587
11	-0.250	-0.233	-.017	1.22
12	0.251	0.257	-.006	1.46
13	0.502	0.509	-.007	0.787
14	0.748	0.754	-.008	0.417
15	0.998	1.010	-.012	0.395
16	1.249	1.269	-.02	0.253
17	1.499	1.509	-.01	0.218
18	2.000	2.015	-.015	0.16
19	2.502	2.512	-.01	0.133
20	2.999	3.010	-.011	0.16
21	3.499	3.526	-.027	0.133
22	4.001	3.998	.003	0.09
23	4.502	4.501	.001	0.083
24	4.999	4.998	.001	0.066

It is the authors belief that the errors in Table I, although quite small, may be due to the normal frequency excursions of the power system; and in fact the measurement accuracy of the algorithm presented here is much greater than that indicated by Table I.

The second experiment consisted of measuring the positive sequence voltage phasors at two ends of a transmission line, while the generator at one end was going through power swings. The experimental set-up is shown in Figure 7.

The analog input data was sampled at sampling pulses generated by a common sampling clock. The positive sequence voltage phasors from the two ends were computed by a single SCDR computer, although in a field installation two separate SCDR's (and also two separate

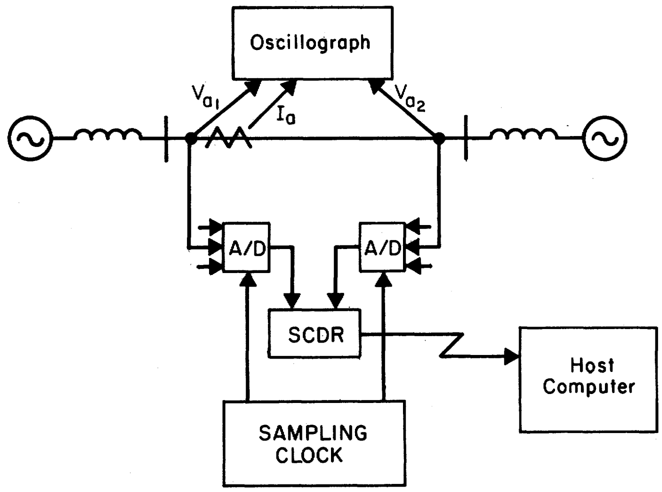

figure 7
Power Swing Data Collection

sampling clocks) would be needed. The phasor samples were transmitted to the host computer for off-line processing. The observed phase $\underline{a}$ voltages from the two ends of the line and phase $\underline{a}$ current in the line are shown in Figure 8(a). Figure 8(b) shows the calculated phase angle difference between the positive sequence voltages of two ends of the line and the computed frequency at each end.

In general, regression relations for computing $\dot{\psi}$ and $\ddot{\psi}$ can be developed from observed values of ψ. However, in practice it has been found adequate to use simple averages over a data window to estimate these quantities:

$$\dot{\psi}_0 = f_0 \approx \frac{1}{2N} \sum_{k=-N}^{N} \frac{\psi_{k+1} - \psi_k}{T_0} \tag{43}$$

and

$$\ddot{\psi}_0 = \dot{f}_0 = \frac{1}{2M} \sum_{k=-M}^{M} \frac{f_{k+1} - f_k}{T_0} \tag{44}$$

where T_0 is the sampling interval, and the window for computing f_0 and $\dot{f}_0$ is (2N+1) and (2M+1) samples respectively.

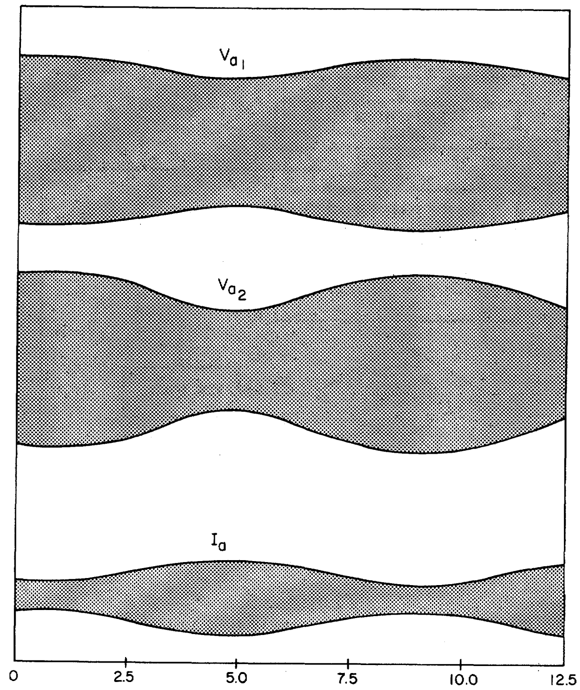

figure 8a
Power Swing Oscillographs

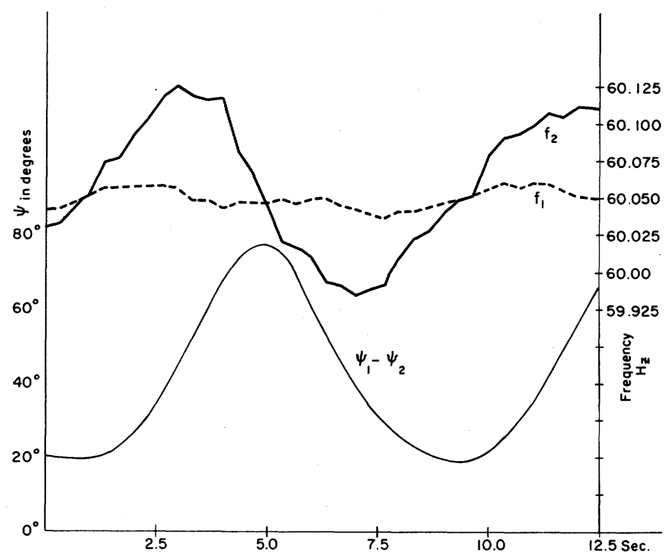

figure 8b
Calculated Frequency and Phase Angle

CONSIDERATIONS FOR FIELD INSTALLATION

Experiments similar to those described above will be performed in the field on AEP system in the coming months. Two Symmetrical Component Distance Relays will be installed at two substations at the two ends of a transmission line. Computation of local phase angles, frequency and rate of change of frequency would be carried out as described above. However, an attempt will also be made to synchronize the sampling clocks at the two substations so that the phase angles measured at the two substations will be with respect to a common reference. One scheme capable of achieving this coordination has been described in Reference [1]. The authors have started a preliminary investigation of the possibility of using a commercially available WWVB receiver as the source for the sampling clock synchronization. Certain on-line applications of phase angle measurement may require high accuracy in the synchronization of sampling clocks. The forthcoming field trials at AEP will be directed towards determining the limits of accuracy that can be achieved with a commercially available WWVB receiver. Other schemes for synchronization of sampling clocks will also be investigated. [9]

CONCLUSIONS

1) An algorithm for measuring phase angle and frequency of ac power system signals has been described. The ac signals are also filtered by the algorithm.
2) The measurement of positive sequence voltage motivated by developments in the digital relaying field is much more useful than the measurements made on one phase of a three phase ac power system.
3) This technique can be implemented as a stand-alone computer based measurement system, or can be made part of a Symmetrical Component Distance Relay used for transmission line protection.
4) The technique of phase angle measurement presented here is not dependent upon zero-crossing detection of a waveform, and hence is not subject to errors created by harmonic components.
5) The laboratory experiments performed on the AEP power system simulator have confirmed the findings of the paper.
6) Making local computations (in a sub-station) which have significance for system-wide estimation and control procedures is a concept which has attracted the interest of power system engineers for a number of years. Techniques presented here may be viewed as a step in a direction of developing one element of a distributed-processing hierarchical computer system for real-time monitoring and control of power systems.

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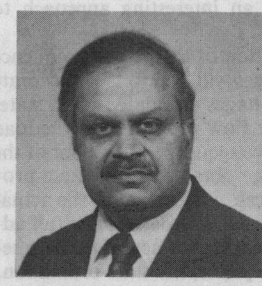
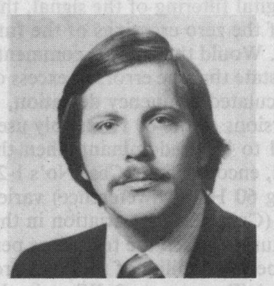
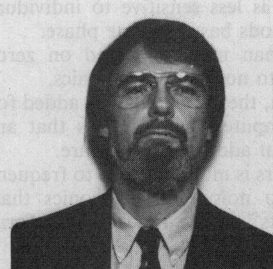
BIOGRAPHIES



Arun G. Phadke, Professor, Electrical Engineering Department, VPI&SU, Blacksburg, VA. Was a System Engineer with Allis-Chalmers Company (Milwaukee, Wis.) from 1963-67 active in the design and development of the AC/DC Simulator Laboratory at the University of Wisconsin, Madison. Was Assistant Professor of Electrical Engineering at the University of Wisconsin, Madison from 1967-69. Joined the American Electric Power Service Corporation in New York in 1969. His responsibilities at AEP as a Consulting Engineer included the management of the Substation Computer Project. He was a Visiting Professor at VPI&SU during 1978-79 on a leave of absence from AEP. He joined VPI&SU permanently in January 1982.



Mark G. Adamiak was born in Teaneck, New Jersey on January 22, 1953. He received his B.S. and Master of Engineering degrees in Electrical Engineering from Cornell University in 1975 and 1976, respectively. In the fall of 1973, and the summers of 1974 and 1975 he worked as a Co-op with AEP and was involved in the building and testing of the AEP A.C. simulator and digital simulation of the Symmetrical Component Distance Relay algorithm. He is presently an Electrical Engineer with AEP in the System Protection and Control Section working exclusively on the substation computer relaying project. He is also presently studying for his M.S.E.E. degree at Polytechnic Institute of New York. Mark is a member of Eta Kappa Nu, Kappa Delta Rho, and IEEE.



James S. Thorp (SM'80) was born in Kansas City, Missouri on February 7, 1937. He received the B.E.E. degree with distinction and the Ph.D. degree both from Cornell University Ithaca, New York in 1959 and 1962 respectively. In 1962 he joined the faculty there, where he is currently a Professor of Electrical Engineering. In 1969-70 he was a member of the Technical Staff, TRW Systems Washington Operations. His other industrial experience includes consulting for the General Electric Company, Light Military Electronics, Ithaca, N.Y. and Advanced Avionics Divison, Johnson City, New York. In 1976-77 he was a Faculty Intern at the American Electric Power Service Corporation, New York, where he is currently a consultant. He is a member of Tau Beta Pi, Eta Kappa Nu and Sigma Xi.

Discussion

R. J. Marttila (Ontario Hydro, Toronto, Canada): The authors have presented a very detailed description of an interesting approach to tracking power system quantities.

The authors make the point that derivation of the positive sequence voltage from the power system phase voltages will lead to more accurate correspondence with positive sequence voltages calculated from state-estimation, load flow, and optimal power flow programs. If the load flow and the other programs use the positive sequence equivalent of the power system, the positive sequence voltage obtained from these programs will not, in the presence of unbalances, correspond to the actual positive sequence voltage in the power system. In order to take full advantage of the improved capability of measuring the actual positive sequence system voltages, as proposed in the paper, the state-estimation, load flow and optimal power flow programs should have the capability of correctly representing unbalance effects in the power system. Would the authors comment?

The authors make a comparison of the relative quality of the bus voltage phase angle measurement as achieved by the digital approach discussed in the paper and by the detection of zero-crossings of the voltage wave. If any non-60 Hz components which could influence the accuracy of the measurement were filtered out from the signal, fundamentally it seems that accuracies comparable to the digital approach should be achievable with the detection of the zero crossings. The digital approach does provide more flexibility and convenience in the implementation, but ultimately, after digital filtering of the signal, this approach also depends on correctness of the zero crossings of the fundamental 60 Hz signal for correct results. Would the authors comment?

In the context of Table I, the authors state that the errors in excess of the expected between the actual and calculated frequency deviation, is probably due to normal frequency excursions of the 60 Hz supply used in the tests. If this source is considered to be predominant, then the results suggest that during the test period, encompassing Case No's 1-24 in Table I, the supply frequency (taking 60 Hz as a reference) varied from 60.012 Hz (Case #1) to 59.973 Hz (Case #21). Is variation in this range expected over the period of time during which the tests were performed? Could another source of error be the stability of the clock frequency (720 Hz ± variation) for the A/D converter? What is the specified stability of the clock to achieve the desired accuracy?

Manuscript received December 22, 1982.

William Premerlani (GE CR&D, Schenectady, NY): The method for measuring frequency and rate-of-change of frequency with a microcomputer described by the authors is one of the most accurate methods described to date. It becomes most sensitive as the frequency approaches the frequency assumed for establishing the sampling rate. As the authors point out, the method bears a "striking resemblance" to the "principle of a power system synchroscope." The advantages of the scheme over other methods include:

1. Using all three phase voltages, it is less sensitive to individual phase voltage variations than methods based on one phase.
2. Using more signal information than methods based on zero-crossing times, it is more immune to noise and harmonics.
3. Because it is based on sampled data, the method can be added for low incremental cost to microcomputer-based devices that are performing other functions, without additional hardware.
4. The method described by the authors is more sensitive to frequency deviation and less sensitive to noise and harmonics than methods based on "leakage effects" in the Fourier Transform calculation.

I believe there is a term missing from the equation given for the rotation of the phasor. It can be found as follows.

The phasor estimate of the signal at time sample r is given by:

$$\bar{X}^{(r)} = \frac{2}{N} \sum_{k=r-N}^{r-1} x(k\Delta t)\, e^{-j\frac{2\pi k}{N}} \tag{1}$$

ΔT = time between samples

A real valued sinusoidal signal can be expressed as:

$$x = \mathrm{Real}(\bar{X}e^{j2\pi ft}) = \frac{\bar{X}e^{j2\pi ft} + \bar{X}^{*}e^{-j2\pi ft}}{2} \tag{2}$$

X = phasor value of x
f = signal frequency
$*$ denotes complex conjugate

Substituting Eq. 2 into Eq. 1, the following is obtained:

$$\bar{X}^{(r)}_{60+\Delta f} = \frac{\bar{X}}{N} e^{+j2\pi(\frac{r}{N} - 1)\frac{f}{60}} \sum_{k=0}^{N-1} e^{j\frac{2\pi k}{N}(\frac{\Delta f}{60})} + \frac{\bar{X}}{N} e^{-j2\pi(\frac{r}{N} - 1)\frac{f}{60}} \sum_{k=0}^{N-1} e^{-j\frac{2\pi k}{N}(\frac{\Delta f}{60} + 2)} \tag{3}$$

Eq. 3 can be simplified using the following identity:

$$\sum_{i=0}^{N-1} (e^{j\theta})^{i} = \frac{\sin\frac{N\theta}{2}}{\sin\frac{\theta}{2}}\, e^{j(N-1)\frac{\theta}{2}} \tag{4}$$

Using Eq. 4 in Eq. 3, the following is obtained:

$$\bar{X}^{(r)}_{60+\Delta f} = \frac{\bar{X}}{N} \frac{\sin\pi\frac{\Delta f}{60}}{\sin\frac{\pi\Delta f}{N60}} e^{-j\pi\frac{\Delta f}{60}(1 + \frac{1}{N})} e^{j\frac{\Delta f}{60}\frac{2\pi}{N}r} + \frac{\bar{X}^{*}}{N} \frac{\sin\pi\frac{\Delta f}{60}}{\sin(\frac{2\pi}{N} + \frac{\pi}{N}\frac{\Delta f}{60})} e^{j\pi(1 + \frac{1}{N})\frac{\Delta f}{60}} e^{j\frac{2\pi}{N}} e^{-j\frac{\Delta f}{60}\frac{2\pi}{N}r} \tag{5}$$

Let $X^{(0)}_{60}$ be denoted:

$$\bar{X}^{(0)}_{60} = \bar{X} e^{-j\pi\frac{\Delta f}{60}(1 + \frac{1}{N})} \tag{6}$$

Then Eq. 5 becomes:

$$\bar{X}^{(r)}_{60+\Delta f} = \bar{X}^{(0)}_{60} \frac{\sin\frac{\Delta f}{60}\pi}{N\sin\frac{\Delta f}{60}\pi} e^{j\frac{\Delta f}{60}\frac{2\pi}{N}r} + \bar{X}^{*(0)}_{60} \frac{\sin\frac{\Delta f}{60}\pi}{N\sin(\frac{\Delta f}{60}\frac{\pi}{N} + \frac{2\pi}{N})} e^{j\frac{2\pi}{N}} e^{-j\frac{\Delta f}{60}\frac{2\pi}{N}r} \tag{7}$$

It can be readily verified that this is the equation of an ellipse in the complex plane. The orientation of the ellipse depends on the phase angle of $X^{(0)}_{60}$. For small frequency deviations, the second term of the equation vanishes, and the equation reduces to that of a circle. As the frequency deviation increases, the eccentricity of the ellipse increases. Fortunately, the second term in the equation is not very large for frequencies near 60 Hz. Would the authors care to comment on the effect of this term on the overall accuracy of the computed frequency using the method they described?

Manuscript received August 2, 1982.

S. Zocholl (Brown Boveri Electric, Inc.): The authors have presented a useful and novel digital technique producing vector rotation which identifies frequency change.

The authors restrict the sampling rate to 720 hertz to conveniently obtain symmetrical component values. It should be noted at a slower sampling rate, say for example, 240 hertz; the symmetrical components may be obtained for a small software burden, using a rotation matrix.

In addition, the slower rate requires less bandwidth.

Regarding the introduction, the authors imply that harmonics effect the zero crossings when measuring frequency. Harmonics have no effect when measured over a full period.

Manuscript received August 4, 1982.

Adly A. Girgis (North Carolina State University, Raleigh, NC): The authors are to be complimented for their application of the discrete Fourier transform in measuring the frequency and its rate of change at a local bus. We have the following comments and questions for the authors' consideration.

Most of the time, engineers are interested in the discrete Fourier transform (DFT) only because it approximates the continuous Fourier transform (CFT). Most of the problems in using the discrete Fourier transform (DFT) are caused by neglecting of what this approximation involves [A]. When the continuous Fourier transform (CFT) is used to analyze a constant-amplitide waveform of a single frequency f, the result is a phasor of a constant amplitude rotating at f cycles/sec. Therefore, when the frequency of the input signal is $60 \pm \Delta f$ Hz, the resultant phasor of the CFT turns an angle of $\pm 2\pi\Delta f$ radians per second relative to the phasor of a 60 Hz waveform.

When the discrete Fourier Transform is used with N points in a window of 1/60 sec, it is assumed that the signal to be analyzed has a constant amplitude and is periodic in a period of 1/60 sec. Any deviation from this assumption will cause errors in calculating the magnitude and the phase angle of the fundamental frequency of the of the DFT (60 Hz component).

Therefore, the error in the phase angle of the fundamental frequency component is function of many parameters, some of these parameters are:

(i) magnitude of frequency deviation $|\Delta f|$
(ii) sign of frequency deviation (positive or negative)
(iii) starting point of the window
(iv) the presence of nonperiodic functions
(v) the larger frequency deviations of the different harmonics ($n\Delta f$ for the nth harmonic), etc.

Aside from the presence of a nonperiodic function or the effect of harmonics, the change in the phase angle of the fundamental frequency component of the DFT is a nonlinear function of the frequency deviation and its sign and the starting point on the window. To show this relation, consider the fundamental frequency component of the DFT (X_1) of a signal x(t) where

$$x(t) = A\sin(2\pi(60 + \Delta f)t + \phi) \tag{D-1}$$

then

$$\bar{X} = \frac{1}{\sqrt{2}} j\bar{X}_1$$

$$= \frac{A}{\sqrt{2}}\frac{2}{N}\left[e^{j\phi}\sum_{k=0}^{N-1} e^{j\frac{\Delta f}{60}\frac{2\pi}{N}k} - e^{-j\phi}\sum_{k=0}^{N-1} e^{-j\left(\frac{4\pi}{N}+\frac{\Delta f}{60}\frac{2\pi}{N}\right)k}\right] \tag{D-2}$$

thus

$$\bar{X} = \bar{X}_{60}\,\frac{\sin\left(\frac{\pi\Delta f}{60}\right)}{N\sin\left(\frac{\Delta f}{60}\frac{\pi}{N}\right)}\, e^{j\Theta_0(\Delta f)} - \bar{X}^*_{60}\, e^{j\frac{2\pi}{N}}\, \frac{\sin\left(\frac{\pi\Delta f}{60}\right)}{N\sin\left(\frac{2\pi}{N}+\frac{\pi}{N}\frac{\Delta f}{60}\right)}\, e^{-j\Theta_0} \tag{D-3}$$

where

$$\Theta_0(\Delta f) = \frac{\pi\Delta f}{60}\left(1 - \frac{1}{N}\right)$$

Eq. (D-3) can be rewritten in the form

$$\bar{X} = \bar{X}_{60}\, F_1(\Delta f)\, e^{j\Theta_0(\Delta f)} - \bar{X}^*_{60}\, e^{j\frac{2\pi}{N}}\, G_1(\Delta f)\, e^{-j\Theta_0(\Delta f)} \tag{D-4}$$

where (for Δf within 5 Hz):

$$F_1(\Delta f) \simeq 1.0$$

$$G_1(\Delta f) = \sin\left(\frac{\pi\Delta f}{60}\right)/N\left(\sin\left(\frac{2\pi}{N} + \frac{\Delta f}{60}\frac{\pi}{N}\right)\right)$$

or

$$G_1(\Delta f) \simeq \left(\pi\frac{\Delta f}{60}\right)/\left(N\sin\frac{2\pi}{N}\right) \tag{D-5}$$

As the change in the phase angle of the fundamental frequency component is considered to be $\frac{2\pi\Delta f}{60}$ radians per cycle of the 60 Hz, the percentage error due to $G_1(\Delta f)$ varies between $-\frac{100}{2N\sin(2\pi/N)}$ and $+\frac{100}{2N\sin(2\pi/N)}$. For $N = 12$, this error will be between -8.3% to $+8.3\%$ depending on the sign of Δf and the starting point of the DFT window. That is in addition to the error due to the initial angle $\theta_0(\Delta f)$. The error due to $\theta_0(\Delta f)$ will not be eliminated by choosing an arbitrary angle θ in the DFT (the angle θ described in Eqs. 14 - 18). That is simply because the arbitrary angle is constant, but the angle $\theta_0(\Delta f)$ is function of the frequency deviation.

Also the effect of the second term of equation (D-4) will not be eliminated by considering the positive sequence voltage. To simplify the mathematical proof of this statement, consider the fundamental frequency component of a sinewave, x(t), and then for a cosinewave, y(t).

$$x(t) = A\sin(2\pi(f + \Delta f)t) \tag{D-6}$$

$$y(t) = A\cos(2\pi(f + \Delta f)t) \tag{D-7}$$

It can be shown from Eq. (D-2 - D-4) that

$$\bar{Y}_1 = F_1(\Delta f)\, e^{j\Theta_0(\Delta f)} + e^{j\frac{2\pi}{N}}\, G_1(\Delta f)\, e^{-j\Theta_0(\Delta f)} = \bar{F}_1 + \bar{G}_1 \tag{D-8}$$

and

$$j\bar{X}_1 = F_1(\Delta f)\, e^{j\Theta_0(\Delta f)} - e^{j\frac{2\pi}{N}}\, G_1(\Delta f)\, e^{-j\Theta_0(\Delta f)} = \bar{F}_1 - \bar{G}_1 \tag{D-9}$$

The phasors Y_1 and jX_1 are shown graphically in Fig. (D-1) which clearly indicates that Y_1 will never be equal to jX_1 since Δf is not equal to zero.

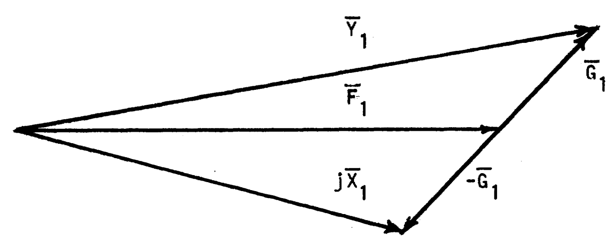


Fig. D-1: Phasor representation of Y_1 and jX_1

Therefore, the voltage of the three phases will not be displaced by 120° in the frequency domain, when there is a frequency deviation Δf.

The measurement time tabulated in Table 1 is calculated according to Eq. 41 ($T = 0.32/\Delta f$), that implies that the measurement time is constant for a constant frequency deviation. However, there is a large difference due to the change of the sign of the frequency deviation. Consider, for example, the frequency deviation of -0.501 and $+.502$, the measurement times are 0.587 and 0.787 sec respectively, which differ by -8.3% and 23% from the values given by Eq. 41. Can this error be considered as a measure of the degree of accuracy of the technique?

The experimental test on a power system model does not show how the power swing condition was experimentally obtained. However, Fig. 8.a implies that the results were obtained by having the two machines of Fig. 7 running at two slightly-different-constant speeds. If that is the case, f_1 and f_2 are supposed to be constants. Examining Fig. 8.b indicates that the frequencies f_1 and f_2 fluctuate according to the fluctuation of the magnitudes of Va_1 and Va_2. Also, comparing Fig. 8.a with Fig. 8.b shows that the fluctuation in f_2 is higher than the fluctuation in

f_1, which appears to be due to the higher fluctuation in Va_2; therefore, can these fluctuations in f_1 and f_2 be considered as a measure of the error due to the change of the signal amplitude?

It is claimed that the method is not affected by harmonics, that is based on the assumption that the frequency of the harmonics is not affected by the frequency deviation of the fundamental. But when the frequency of the fundamental component changes by Δf, the frequency of the nth harmonic changes by $n\Delta f$.

The fundamental frequency (the 60 Hz component) of a signal of frequency $n(60 \pm \Delta f)$ will be a phasor quantity whose magnitude and phase angle are functions of the frequency deviation (Δf) of the fundamental. It should be also noted that the effect of this harmonic will not be diminated by considering the positive sequence voltage. The reason is that the fundamental frequency components of the nth harmonic in the three phases will not be displaced by an angle of $2\pi \frac{n}{3}$.

The technique will be affected by any nonperiodic signal that may be present in the waveform such as dc offset or voltage spikes. Do the authors intend to use an analog filter to filter such nonperiodic functions?

The leakage coefficient method for detecting the frequency deviation is based on the fact that due to a frequency deviation of the 60 Hz waveform by $\pm \Delta f$, the magnitude of any of the harmonics in the DFT is not zero. We found that the sum of the magnitudes of the harmonics in the DFT is linearly related to the magnitude of the frequency deviation, provided that the waveform to be analyzed is a sinewave [2, A), therefore, a zero crossing detector is used to have a fixed starting point on the waveform to be analyzed. As the sum of DFT components will be also affected by the presence of non 60 Hz component, the first step in the leakage coefficient scheme is to band-pass filter the waveform. The band-pass filter used in a 4-pole Butterworth filter with a center frequency of 60 Hz and a bandwidth of 30 Hz [B, C]. The leakage coefficient technique is not affected by the change of the amplitude of the waveform to be analyzed. Furthermore, any frequency deviation is calculated in two cycles (33 msec). The change of the angle of DFT fundamental component is also used in the leakage coefficient method to detect only the sign of the frequency deviation (position or negative).

Finally, I congratulate the authors for an interesting paper.

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[C] F. Ham and Adly A. Girgis, "Measurement of Power Frequency Fluctuations Using FFT", to be published.

Manuscript received August 13, 1982.

J. G. Gilbert and **N. P. Marino** (Pennsylvania Power & Light Company, Allentown, PA): The authors have made a useful contribution to the art of computer protection and control of power systems. We would appreciate having the author's thoughts on the following to clarify concepts they have proposed:

1. A fixed frequency sampling clock is used with the SCDR computer described by the authors. Had a frequency tracking sampler been employed, power system frequency could not be measured in the manner described in this paper. What issues lead to the author's choice of a fixed frequency sampling clock? Do the authors believe it worthwhile and practical to compensate results of frequency sensitive calculations based on computed power system frequency?
2. The authors assume four or more raw phase angle measurements would be used to produce one smoothed frequency result. Did the authors consider using phase angle measurements made from a longer data window? Would the authors discuss benefits of smoothing several raw phase angle measurements instead of using a correspondingly longer data window?
3. Fig. 8b graphically shows that precautions are required to prevent misoperation of load shedding schemes during power swings. Time delay has been used to prevent misoperation of conventional relay schemes. Have the authors investigated any alternatives to intentional time delay for this application?

We believe some of the issues raised by this paper point to the need for early establishment of guidelines and standards. A nonexhaustive list of subjects which comes to mind is: establishment of standard sampling rate(s); selection of fixed frequency versus frequency tracking sampling clocks; standard electrical interfaces to digital protection subsystems; man-machine interface standards; identification, modularity, and residency of standard protection and control functions; standards for test and calibration of digital protection subsystems; etc. This list can be made much longer. Early development of guidelines and standards for issues such as these could prevent many future concerns and difficulties.

Manuscript received August 2, 1982.

A. G. Phadke, J. S. Thorp, and **M. G. Adamiak:** We thank the discussors for the interest they have shown in our paper.

Mr. Martilla observes correctly that to form an entirely consistent model of the power system, its measurements, the model and the computation procedure used should all use a positive-sequence frame of reference. In most cases, the positive sequence network model used in load flow or state estimation programs simply neglects the negative and zero sequence networks and their coupling to the positive sequence network. A model which included the coupled network would be the most accurate model, and when combined with sequence voltage measurements would produce an accurate representation of the system. In our view, measuring the positive sequence voltage on the positive sequence network model (neglecting the couplings) is a step in the direction of improving model consistency. In many cases, this improvement may be all that is called for. We believe this procedure would be better than using phase voltage measurements with the positive sequence network model. Perhaps additional investigation in this subject is justified where system unbalances are a significant factor.

Another point raised by Mr. Martilla discusses the efficacy of using zero crossing information for frequency measurement. Analog filters, at least in their practical realizations are not as sharp as digital (Fourier type) filters in rejecting harmonics. Further, as pointed out by Mr. Premerlani in his discussion, since the Fourier type technique uses more data (including the non-zero sample values), it forms a more reliable estimate of the frequency. Ultimately, if the analog filters used were able to produce a pure fundamental frequency component, then its zero-crossing frequency measurement would be as accurate as that produced by the digital technique. However, such an analog filter would not function when the power system frequency changes from the nominal power frequency.

The last point made by Mr. Martilla concerns the frequency excursions observed in our area. Although it was not mentioned in the paper, the data collection was carried out over several days, and consequently the observed frequency excursions are well within normal variations of the power system frequency. The crystal oscillator clock used had a stability of 10 parts per million. It seems to us that such accuracies being readily available, the stability of the clock should be of no concern in the present context.

Mr. Zocholl is quite right in his statement that a sampling rate of 240 Hz can also be used to produce symmetrical components. In fact, almost any sampling rate would be acceptable, although the computational burden associated with the transformation from phasors to symmetrical components depends upon the sampling rate chosen. A sampling rate which is a multiple of 180 Hz has particularly advantageous properties, as pointed out in our Ref. (4). Also, as the digital relays we have developed use a sampling rate of 720 Hz, the technique presented allows a computation of frequency with little overhead. We are not sure about the import of Mr. Socholl's comment about the bandwidth. It seems to us that in terms of presently available analog components and digital computers, there is not much difference between bandwidths associated with sampling rates of 180 Hz, 240 Hz or 720 Hz.

Mr. Zocholl is quite right in pointing out that a stationary harmonic content does not affect the zero crossing of a waveform. However, in our observations, we have noted a constant movement of the harmonic components, and consequently the zero-crossing does change due to changing harmonic components in system voltage waveforms. It should be noted that several successive zero-crossing points must be used to create a single frequency measurement.

Mr. Gilbert and Mr. Marino have correctly pointed out the reasons for using sampling clocks that are independent of power system frequency. Without such clocks, frequency measurements would not be possible. As a practical matter, such clocks are also much simpler to construct than clocks which are synchronized to the power system waveform.

In some cases, relaying parameters are dependent upon frequency. Although the present authors' technique using symmetrical components is immune to power system frequency deviations, other impedance relays may depend upon the frequency. Since the frequency is known (obtained by using the technique described in this paper), the frequency correction could be applied to the relay characteristic where appropriate. Individual cases must be considered separately to determine specific frequency compensation to be applied.

In general, either raw phase angle measurements or averaged phase angle measurements could be used to calculate the frequency, as long as at least two phase angle measurements are used at sufficiently separated instants of time. It should be pointed out that averaging of data from a rotating phasor could lead to serious errors if the averaging is done over a large angle of rotation. To take an extreme example, the average of two phasors which are obtained from a rotating phasor when the amount of rotation is 180° is zero, a result which is of no use in any frequency calculation.

We have not studied the problem of load shedding relays. This clearly is an application problem. If engineering analysis shows that higher order derivatives of frequency should be used as inputs to the load shedding relay, then it would be a simple matter to compute the necessary frequency derivatives by using the regression technique outlined in the paper.

We agree with Messrs. Gilbert and Marino that there is a great need for standardization in the areas they mention. Perhaps, in time, such standards will be developed by technical subcommittees and working groups of IEEE.

Mr. Premerlani has given an excellent summary of the advantages of a frequency measurement technique based on recursive Fourier Transforms. We are also grateful to him and to Mr. Girgis for pointing out that Eq. (31) of our paper is an approximation. The analysis given by Messrs. Premerlani and Girgis gives the correct form, and it is instructive to study the effect of this approximation. Taking Eq. (7) of Mr. Premerlani (which is essentially the same as Eq. (D-4) of Mr. Girgis) it should be noted that the phasor at (60 + Δf) inscribes an ellipse in the complex plane. The equation for the ellipse is of the type

$$\bar{X} = A\,e^{j\Delta\omega t} + xA^{*}\,e^{j\frac{2\pi}{N}}\,e^{-j\Delta\omega t}$$

where x is a scale factor given by

$$x = \frac{\sin\frac{\pi\Delta f}{60N}}{\sin\left(\frac{\pi\Delta f}{60N} + \frac{2\pi}{N}\right)}$$

The two phasors A and xA^* rotate in opposite directions in the complex plane, and at $t=0$ the resultant phasor is

$$\bar{X}\ (t=0) = A + xA^{*}\,e^{j\frac{2\pi}{N}}$$

This relationship is illustrated in Fig. 9 for an arbitrary phasor A using a somewhat magnified value for x.

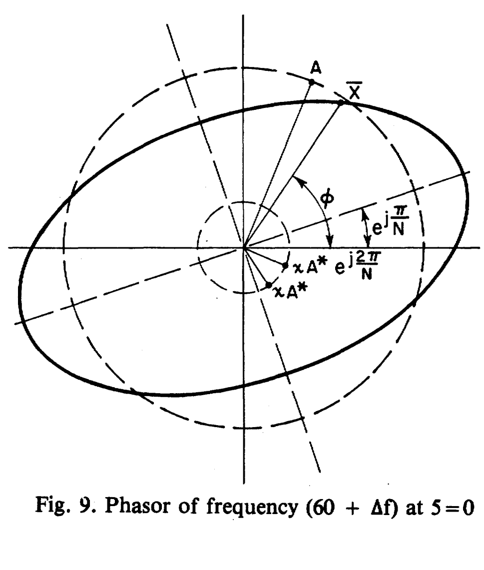

Fig. 9. Phasor of frequency (60 + Δf) at 5 = 0

The major axis of the ellipse is at an angle π/N with the real axis, and the minor axis is perpendicular to it. This angle results from the placement of the sampling instants with respect to the time zero point, and does not depend upon $X^{(0)}_{60}$ or any other paramter. Disregarding this shift in angle (or assuming that the sampling instants are evenly placed around $t=0$, leads to an ellipse placed with its major axis along the real axis.

One could calculate the angular velocity of the resultant phasor which describes the ellipse. With some manipulations, it can be shown that the angular velocity $\frac{d\phi}{dt}$ of the phasor $X_{(60+\Delta f)}$ is given by

$$\frac{d\phi}{dt} = \Delta\omega\,\frac{1+\tan^{2}\Delta\omega t}{\frac{1+x}{1-x} + \frac{1-x}{1+x}\tan^{2}\Delta\omega t}$$

Clearly the angular velocity differs from Δω when x is not equal to zero as pointed out by Messrs. Premerlani and Girgis. The variation of $\frac{d\phi}{dt}$ is cyclic around Δω, varying between $\frac{1+x}{1-x}\Delta\omega$ and $\frac{1-x}{1+x}\Delta\omega$ as shown in Fig. 10.

For a frequency difference of 1 Hz, x is 0.00866, and consequently the measured frequency would vary between 60 ± (0.9828) Hz to 60 ± (1.017) Hz depending upon the instant of measurement.

This phenomenon suggests a remedy to the problem: If $\frac{d\phi}{dt}$ is averaged over a half period, the effect of this variation will be removed for small values of Δf. Indeed, if the frequency is to be measured from a single phasor, this averaging procedure should be used.

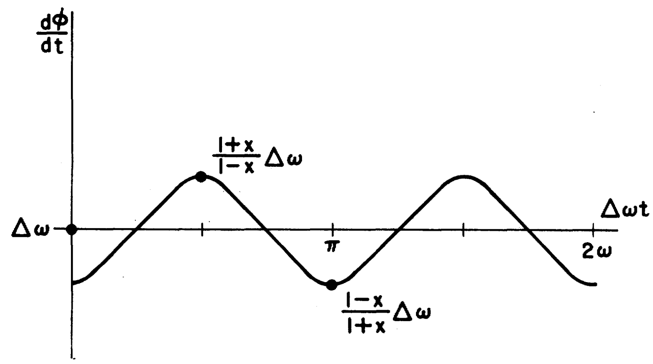

Fig. 10. Variation of computed frequency

However, when positive sequence quantities are used, there is an automatic filtering effect of this variation. We will now consider the positive sequence computation from three phase voltage measurements where the power system frequency is (60 + Δf) Hz. (In a brief conversation at the Summer Power Meeting, we had mentioned this phenomenon to Mr. Girgis. We are not certain whether we explained our idea in sufficient detail at that time. Perhaps this explanation will help answer Mr. Girgis' question on symmetrical components.)

Each of the three phase voltage phasors at frequency (60 + Δf) Hz has two components, one rotating in clockwise direction, and the other in a counterclockwise direction. As the axes of the ellipse are fixed by the sampling instants, they are identical for the three phasors. See Fig. 11: (The axes of the ellipse are drawn coincident with real and imaginary axes for convenience. As pointed out earlier, this choice is immaterial to the present discussion.)

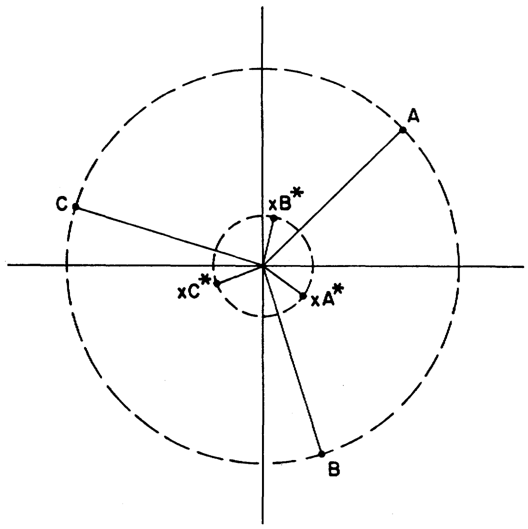

Fig. 11. Positive Sequence Measurement at $(60 + \Delta f)$ Hz

The phasors A, B, C rotate in the clockwise direction, whereas phasors xA^*, xB^* and xC^* rotate in the counterclockwise direction. Note that xA^* is obtained as a mirror image of A in the real axis with a scale factor x, and similarly for xB^* and xC^*. It is clear from Fig. 11 that while A, B, C are positive sequence quantities, xA^*, xB^* and xC^* are negative sequence quantities. Consequently, when the symmetrical component transformation is applied to $(A + xA^*)$, $(B + xB^*)$, $(C + xC^*)$, the positive sequence component of the result will be the same as though it was computed from (A, B, C). The frequency computed from the positive sequence voltage phasor is thus free of the cyclic variation shown in Fig. 10; and reads $\Delta\omega$ correctly. No averaging is necessary, and the correction term omitted from Eq. (31) of the paper is indeed unnecessary.

To answer the questions of Mr. Girgis regarding the experiment, the machines were *not* turned at constant speeds, indeed one machine was oscillated around the synchronous speed. The measured frequency change is the actual frequency change, and is immune to the changes of voltage amplitude.

The measurement times do not follow Eq. (41) of the paper exactly, it being a continuous variable result, whereas the actual measurement time jumps in units of the data window (one cycle). Eq. (41) should be viewed as a general guideline for the kind of meaurement times to be expected.

The harmonic effect mentioned by Mr. Girgis is very interesting, but we have not studied it in any detail. Perhaps such a study should be made. However, we would make the observation that the leakage effect method will fail completely in the presence of harmonics, in fact we find this to be one of the main drawbacks of the leakage effect methods.

We would like to conclude by saying that the method presented in the paper is extremely sensitive and accurate for small frequency deviations. Since this is the nature of frequency excursions in a power system, the method is particularly attractive. Also, by using phasors which are computed for digital relaying applications, the method adds frequency measurement function to a relay at no additional burden on the relaying processor.

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